

More details on the QRT map as an  
automorphism of a rational elliptic surface

Professor J.J. Duistermaat

Mathematisch Instituut, Universiteit Utrecht

The Netherlands

Saturday 15 July 2006

AMSI Seminar Room

111 Barry Street, Carlton

The elliptic curves, on each of which the QRT map acts as a translation, form a pencil of biquadratic curves in the Cartesian product of two complex projective lines. The natural domain of definition of the QRT map is a smooth complex projective (hence compact) surface  $S$ , obtained by successively blowing up  $P^1 \times P^1$  in the 8 base points of the pencil of biquadratic curves. The QRT map is an automorphism of  $S$ , a globally defined complex analytic diffeomorphism from  $S$  onto  $S$ . The homology classes of real two-dimensional cycles in  $S$  form a lattice  $L$  of rank 10, and the automorphisms of  $S$  are determined by their action on  $L$ . We describe the action of the QRT map on  $L$ , and obtain as an application that for the generic QRT map the number of biquadratic curves, on which the QRT map is periodic with period  $k$ , is equal to  $k^2 - 1$ . Here “generic” means that the degree of the rational expressions which appear in the formula for the QRT map does not drop because of common factors in the numerator and the denominator. We also pay attention to some non-generic cases. The QRT surface  $S$  is a so-called rational elliptic surface, and we determine which rational elliptic surfaces occur as QRT surfaces.

## Program

9.00-9.45	1st lecture
9.45-10.00	break
10.00-10.45	2nd lecture
10.45-11.15	Tea and Coffee
11.15-12.00	3rd lecture
12.00-12.15	break
12.15-13.15	4th lecture
13.15	lunch

A videorecording will be made of the lectures and interested participants should contact the organisers regarding a DVD copy. This meeting is funded by the ARC Centre of Excellence for Mathematics and Statistics of Complex Systems and supported by the Australian Mathematical Sciences Institute.

Organisers:

Professor R. Quispel ([R.Quispel@latrobe.edu.au](mailto:R.Quispel@latrobe.edu.au))

Dr. N.S. Witte ([N.Witte@ms.unimelb.edu.au](mailto:N.Witte@ms.unimelb.edu.au))