Errata for the Book *Strongly Elliptic Systems and Boundary Integral Equations*

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27(-6) The statement of Theorem 2.14 should read "In any complete metric space X, ...".

28(-10) Here, it would be better to say "any (bounded) linear operator".

60(5) One usually proves $\omega_p(t,u) \to 0$ first for $u \in C^0_{\mathrm{comp}}(\Omega)$ and then uses density of $C^0_{\mathrm{comp}}(\Omega)$ in $L_p(\mathbb{R}^n)$ $(1 \le p < \infty)$.

64(7) Note that the constant C depends on α but not on ϵ.

66(-7,-10) "largest relatively closed set" should be "smallest relatively closed set".

71(-2) Here, continuity of $\hat{u}$ is not needed.

72(8) $x^\alpha \partial_j^\beta \phi(x)$ should be $x^\alpha \partial^\beta \phi_j(x)$.

73(-10) $\mathcal{F}_{\xi \to x}$ should be $\mathcal{F}_{x \to \xi}$.

78(2) The second inclusion is true for $s \ge 0$, although $\|u|_\Omega\|_{H^s_0(\Omega)} \le \|u\|_{\widetilde{H}^s(\Omega)}$ holds for all $s \in \mathbb{R}$. If $s < -1/2$ then Lemma 3.39 shows that $u \mapsto u|_\Omega$ is not injective, even for smooth Ω.

79(7) $\mathcal{D}(\mathbb{R}^n \setminus \Omega)$ should be $\mathcal{D}(\mathbb{R}^n \setminus \overline{\Omega})$.

89(-7) Equation (3.26) should read

$$\Omega = \{\, x \in \mathbb{R}^n : x_n < \zeta(x') \text{ and } x' = (x_1, \ldots, x_{n-1}) \in \mathbb{R}^{n-1} \,\}.$$

99(-7) I omitted the definition of the space

$$\mathcal{D}(\Gamma) = \{\, u : u = U|_\Gamma \text{ for some } U \in \mathcal{D}(\mathbb{R}^n) \,\}.$$

100(6) The definition of γ should read $\gamma u(x') = u(x', 0)$.

111(13) [Thanks to Paolo Gatto 15-03-2017.] In problem 3.18, $H^s(\Omega)$ should be $W^s(\Omega)$.

112(-9) $k \geq 0$ should be $k \geq 1$.

115(6) [Thanks to Anupam Pal Choudhury, 05-12-2014.] The assumptions that $u \in H^2(\Omega)^m$ and $v \in H^1(\Omega)^m$ are not sufficient to ensure $w = (\mathcal{B}_j u)^* v \in H^1(\Omega)^m$, since in general the product to two H^1 functions is not an H^1 function. Instead, we should assume in the first instance that v is the restriction to Ω of (say) a C^1 function with compact support in $\mathbb{R}^n$, so that $w \in H^1(\Omega)$. Lemma 4.1 then follows because such vs are dense in $H^1(\Omega)^m$, and the first Green identity is continuous in v with respect to the topology of $H^1(\Omega)^m$.

116(-8) The conditions (4.6) are sufficient to ensure $\mathcal{P}^* = \mathcal{P}$, but they are not necessary. It would be better to change the definition (4.1) of $\mathcal{P}$ to

$$\mathcal{P} = -\sum_{j=1}^n \sum_{k=1}^n \partial_j (A_{jk}\partial_k u - A_j u) + \sum_{j=1}^n A_j \partial_j u + Au.$$

Anything of this form can be written in the form (4.1), and vice versa, since we assume all coefficients are smooth. The advantage of the new definition is that

$$\mathcal{P}^* = -\sum_{j=1}^n \sum_{k=1}^n \partial_j (A_{kj}^*\partial_k u + A_j^* u) - \sum_{j=1}^n A_j^* \partial_j u + A^* u,$$

and so $\mathcal{P}^* = \mathcal{P}$ iff

$$A_{kj}^* = A_{jk}, \quad A_j^* = -A_j, \quad A^* = A.$$

Moreover, the conormal derivatives for $\mathcal{P}$ and $\mathcal{P}^*$ now look more symmetric:

$$\mathcal{B}_\nu u = \sum_{j=1}^n \nu_j \biggl(\sum_{k=1}^n A_{jk}\partial_k u - A_j u \biggr)$$

and

$$\widetilde{\mathcal{B}}_\nu u = \sum_{j=1}^{n} \nu_j \biggl(\sum_{k=1}^{n} A_{kj}^* \partial_k u + A_j^* u \biggr).$$

Moreover, the conormal derivative for $(\mathcal{P}^*)^*$ becomes the same as that for $\mathcal{P}$, i.e., $\mathcal{B}_\nu$.

151 (3) (Noticed by Sergey Mikhailov.) The definition of g must include a cutoff function $\chi \in C_{\text{comp}}^\infty(\mathbb{R})$,

$$g(x) = \chi(x_n)\gamma u\bigl(x', \zeta(x')\bigr),$$

with $\chi(x_n) = 1$ for all x in a neighbourhood of Γ. Otherwise, g will not belong to $H^1(\Omega)^m$.

213 (7) (Noticed by Abderrahmane Bendali.) We can put $u = \delta_x$ only if $m = 1$. In the vector-valued case, we instead put $u = \delta_x \eta$ for an arbitrary constant vector $\eta \in \mathbb{C}^m$, so that

$$\mathcal{P}^* G(x, \cdot)^* \eta = \delta_x \eta - K_2(x, \cdot)^* \eta.$$

324 (4) The claim about "equal norms" is false, as noted in the paper

> S. N. Chandler-Wilde, D. P. Hewett and A. Moiola, Interpolation of Hilbert and Sobolev spaces: quantitative estimates and counterexamples, *Mathematika* 61:414–443, 2015, DOI: 10.1112/S0025579314000278