Stratospheric mean residence time and mean age on the tropopause: Connections and implications for observational constraints

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[1] Stratospheric mean residence time $\bar{\tau}$ and mean age on the tropopause Γ_Ω are shown to measure physically distinct aspects of stratospheric transport. Both Γ_Ω and $\bar{\tau}$ are mean transit times through the stratosphere of air that enters through tropopause region Ω_i and exits through region Ω_f, but they represent averages over different populations of fluid elements. The averaging for Γ_Ω is based on the population of fluid elements exiting the stratosphere, while $\bar{\tau}$ is based on the population of $\Omega_i \rightarrow \Omega_f$ fluid elements residing in the stratosphere. Thus, Γ_Ω is the mean age on exit, while $\bar{\tau}$ is the expected mean residence time in the interior of the stratosphere. The physical basis for defining and robustly computing both timescales is the one-way stratosphere-to-troposphere flux of air labeled with the boundary-propagator Green function, $\mathcal{G}$. By re-expressing the boundary-value problem for $\mathcal{G}$ in terms of first-order loss in a tropopause layer with a timescale τ_c in the limit $\tau_c \rightarrow 0$, we show that both $\bar{\tau}$ and Γ_Ω can be obtained as ratios of moments of $\mathcal{G}$ extrapolated to the tropopause. One obtains $\bar{\tau} = \Gamma_\Omega + 2\Delta^2/\Gamma_\Omega$, where Δ quantifies the width of the transit-time distribution. Because the moments of $\mathcal{G}$ can be estimated from the mixing ratio of transient trace gases, it is in principle possible to estimate $\bar{\tau}$ from measurements of two independent transient tracers. The distinctness of $\bar{\tau}$ and Γ_Ω is elucidated using idealized models.

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1. Introduction

[2] Mean transit time from a suitable reference region, also referred to as "mean age," is a widely applied diagnostic for understanding and interpreting stratospheric transport and trace gas distributions (e.g., the reviews by *Waugh and Hall* [2002] and *Plumb* [2002]). A related timescale recently introduced in the context of stratosphere-troposphere exchange is the mean residence time $\bar{\tau}$, which quantifies the mean time between successive tropopause contacts [*Orbe et al.*, 2012]. Mean age is defined as the first moment of the transit-time distribution (sometimes referred to as the "age spectrum" [*Hall and Plumb*, 1994]), while mean residence time is the first moment of the transport-mass distribution derived from the transit-time partitioned one-way flux across the tropopause [*Orbe et al.*, 2012].

[3] Mean age can be defined as the mean transit time since last contact with any region where air is considered to have its age reset to zero. Stratospheric mean age is usually defined to be the mean time since last tropopause contact and is therefore defined to be zero over the entire tropopause, although this boundary condition has been explicitly enforced primarily for idealized analytic [*Neu and Plumb*, 1999; *Hall and Waugh*, 2000] or two-dimensional models [e.g., *Boering et al.*, 1996]. For three-dimensional circulation models it is common to apply the zero boundary condition at the surface and then correct by subtracting the resulting mean age at the tropical tropopause (or a nominal height) to obtain an approximate stratospheric mean age [e.g., *Hall et al.*, 1999; *Eyring et al.*, 2006]. However, this procedure does not yield accurate mean transit times since last tropopause contact in the lower stratosphere, and is primarily a convenient approximation to avoid tracking the tropopause.

[4] In this paper, we refine the boundary conditions on stratospheric mean age so that the mean transit time since last contact with a particular region of the tropopause (e.g., the tropical tropopause) can be quantified not just in the interior of the stratosphere, but also on the tropopause itself

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while still stripping air of its stratospheric identity as soon as it crosses the tropopause. We refer to the resulting stratospheric mean age on the tropopause as the "mean age on exit". We show that for advective-diffusive flow, the stratospheric mean age on exit and the mean residence time measure fundamentally different quantities. We also show that the mean age on exit and the mean residence time can both be expressed as a limit of the ratio of moments of the transit-time distribution. Thus, both timescales are, at least in principle, accessible from observations of transient tracers.

[5] Like much of the existing work on mean age and residence times, the relationship between mean entry-to-exit residence time and mean age on exit that we derive here is an extension of the earlier body of work by *Bolin and Rhode* [1973], *Nir and Lewis* [1975], and *Björkström* [1978]. These authors were among the first to distinguish between the distribution of times to traverse a geophysical fluid reservoir and the distribution of times since entry into the reservoir. Stratospheric residence time has been used to quantify short-lived stratosphere-troposphere exchange events [*Wernli and Bourqui*, 2002; *Stohl et al.*, 2003], but these studies did not consider residence time more broadly as a transport diagnostic or link it to the mean age of air. *Hall and Waugh* [2000] studied the empirical relationship between stratospheric mean age and the mean residence time of tracer injected into the interior of the stratosphere. This interior-to-exit mean residence time should not be confused with the entry-to-exit residence time considered here, although both can be formulated in terms of a suitable mass-to-flux ratio. The general results developed below have, to the best of our knowledge, not been derived previously nor have they been applied to atmospheric transport.

2. Boundary Propagator, Air-Mass Fraction, and Mean Transit Time

[6] Mean transit time or "age" can be defined as the mean time since last contact with any specified region Ω such as, for example, the tropical tropopause, the entire tropopause, the earth's surface, or the tropical troposphere. However, to quantify the transport of stratospheric air, it is most natural to take Ω to be a suitably defined tropopause and to then "bin" or partition stratospheric air according to when and where it last crossed the tropopause. To identify where a fluid element (or simply particle) last crossed the tropopause, we subdivide the tropopause into smaller regions $\Omega_i \subseteq \Omega$. In order to assign a transit time since last Ω_i contact, we label air with a passive tracer $\mathcal{G}$ when it crosses into the stratosphere at Ω_i during a specified time interval $(t', t' + dt')$. It is convenient to refer to the $\mathcal{G}$-labeled air as "(Ω_i, t') air". In order to ensure that every air particle can be assigned a unique last contact region and time, it is important to remove the (Ω_i, t') tracer label on subsequent contact anywhere with the tropopause Ω [*Holzer and Hall*, 2000; *Primeau*, 2005]. Note that if $\Omega_i = \Omega$, the mean time since last Ω_i contact must be zero everywhere on the tropopause, in which case it is clear that a finite mean residence time and zero mean age on the tropopause are distinct quantities [*Orbe et al.*, 2012]. Below we show that if Ω_i is a finite subset of Ω such as the tropical tropopause, then mean age on the tropopause outside of Ω_i is finite, representing the finite transit time from Ω_i to other regions of the tropopause. However, because there is by definition zero Ω_i air on the tropopause outside of Ω_i, the mean age of Ω_i air involves a subtle limit there. We will now develop the equations for (Ω_i, t') air so that this limit, and the connections between mean age on exit and entry-to-exit mean residence time, can straightforwardly be derived.

2.1. Boundary Propagator, $\mathcal{G}$

[7] The mass-fraction of (Ω_i, t') air at $(\mathbf{r}, t)$ is given by $\mathcal{G}(\mathbf{r}, t|\Omega_i, t')dt'$, where the tracer label $\mathcal{G}$ is also the boundary-propagator Green function. The boundary propagator obeys the source-free advection-diffusion equation

$$\partial_t(\rho\mathcal{G}) + \nabla \cdot \mathbf{J} = 0, \tag{1}$$

where $\mathbf{J}$ is the advective-diffusive mass flux of $\mathcal{G}$-labeled air, ρ is the density of air, and $\mathcal{G}$ is subject to the boundary condition [*Holzer and Hall*, 2000]

$$\mathcal{G}(\mathbf{r}_\Omega, t|\Omega_i, t') = \delta(t - t')\Delta_i(\mathbf{r}_\Omega, t) \tag{2}$$

at points $\mathbf{r}_\Omega$ on the tropopause, where $\Delta_i(\mathbf{r}, t) = 1$ if $\mathbf{r} \in \Omega_i$ and $\Delta_i(\mathbf{r}, t) = 0$ otherwise. Note that the time dependence of the mask Δ_i comes from the fact that the tropopause is generally in motion.

[8] For the sake of simplicity and to make the connections between various quantities more transparent, we will specialize from now on to steady flow. In practice this means that our relationships can be applied to ensemble-mean boundary propagators [e.g., *Holzer et al.*, 2003; *Haine et al.*, 2008]. For steady flow, $\mathcal{G}$ depends on time only through the transit time $\xi = t - t'$, so that $\mathcal{G}(\mathbf{r}, t|\Omega_i, t') \to \mathcal{G}(\mathbf{r}|\Omega_i, \xi)$. The development below can straightforwardly be extended to arbitrary time-dependent flow at the expense of more complicated expressions. The boundary propagator $\mathcal{G}$, with the normalization $\sum_{\Omega_i}\int_0^\infty d\xi\mathcal{G} = 1$, generalizes the transit-time distribution to a joint distribution of times ξ and locations Ω_i since last surface contact.

2.2. Air-Mass Fraction, f

[9] The mass fraction $f(\mathbf{r}|\Omega_i)$ of Ω_i air at $\mathbf{r}$ regardless of when this air had last Ω_i contact is given by

$$f(\mathbf{r}|\Omega_i) = \int_0^\infty \mathcal{G}(\mathbf{r}|\Omega_i, \xi)d\xi \equiv \langle\mathcal{G}\rangle, \tag{3}$$

where we have introduced angle brackets to denote the integral over all transit times, and in the case of statistically stationary flow, an additional ensemble average. Note that when Ω_i is the entire tropopause, $f = 1$, because all air must have had tropopause contact sometime in the past. However, if Ω_i is a smaller subset of the tropopause, then $0 < f < 1$ in the interior of the stratosphere, $f = 1$ only on Ω_i, and $f = 0$ elsewhere on the tropopause. That $f = 0$ on the complement of Ω_i, which we will denote $\Omega^{ci} = \Omega \setminus \Omega_i$, simply expresses the fact that on Ω^{ci} all air had last contact with Ω^{ci} and none of the air had last contact on Ω_i.

2.3. Mean Transit Time, or Age, Γ

[10] The mean transit time of an air parcel at $\mathbf{r}$ since last contact with Ω_i, denoted by $\Gamma(\mathbf{r}, \Omega_i)$ and also referred to as

the ideal mean age, particularly when $\Omega_i = \Omega$ [*Waugh and Hall*, 2002], is given by

$$\Gamma(\mathbf{r}, \Omega_i) = \frac{1}{f(\mathbf{r}|\Omega_i)} \int_0^\infty \xi \mathcal{G}(\mathbf{r}|\Omega_i, \xi) d\xi = \frac{\langle \xi \mathcal{G} \rangle}{\langle \mathcal{G} \rangle}. \tag{4}$$

[11] Note that for $\mathbf{r} \in \Omega_i$ we have $\mathcal{G}(\mathbf{r}|\Omega_i, \xi) = \delta(\xi)$, and hence $\Gamma(\mathbf{r}_\Omega, \Omega_i) = 0$ because the mean time since last Ω_i contact must be zero on Ω_i. One reasonably expects Ω_i air to have a finite mean transit time from Ω_i to points on Ω^{ci}. However, $\mathcal{G} = 0$ on Ω^{ci} because by definition there is zero Ω_i air on Ω^{ci} and, in particular, $f(\mathbf{r}|\Omega_i) \to 0$ as $\mathbf{r}$ approaches Ω^{ci}. Thus, the mean transit time from Ω_i to tropopause locations outside of Ω_i cannot directly be computed from $\mathcal{G}$ on the tropopause, but must be carefully evaluated as a "zero-over-zero" limit. In the next section, we show that the mean age on the tropopause can be given physical meaning in terms of the one-way flux of Ω_i air across the tropopause. This avoids a "zero-over-zero" limit and therefore allows for more robust computation. The one-way flux is induced by the boundary conditions (2), which hold the mixing ratio $\mathcal{G}$ at zero on the tropopause for $t > t'$ to remove the (Ω_i, t') identity from $\mathcal{G}$-labeled air. This flux also leads very naturally to a distribution of residence times, which allows us to explore the difference between mean age on exit and mean entry-to-exit residence time.

3. One-Way Cross-Tropopause Flux-Density Distribution, $\mathcal{J}$

[12] To simplify the mathematics of the limit involved in evaluating mean age on the tropopause as much as possible, we first recast the boundary value problem for $\mathcal{G}$ in a simpler form. Instead of specifying the boundary condition (2) on Ω, we model the unlabeling of Ω_i air as first-order destruction with a finite timescale τ_c in a layer of non-zero thickness dn just underneath the tropopause. (For our purposes the layer could have any thickness, but we use infinitesimal notation for convenience.) This layer can be considered to be a thickened Ω surface, and we will refer to it as the Ω layer. We denote the resulting mixing ratio of Ω_i air by $\mathcal{G}_c$. If we identify the Ω layer at time t with the mask $\Delta_\Omega(\mathbf{r}, t) = 1$ if $\mathbf{r}$ is in the layer and $\Delta_\Omega(\mathbf{r}, t) = 0$ otherwise, the equation for $\mathcal{G}_c(\mathbf{r}, t|\Omega_i, t')$ may be written as

$$\partial_t(\rho \mathcal{G}_c) + \nabla \cdot \mathbf{J} = -\rho \frac{\Delta_\Omega(\mathbf{r}, t)}{\tau_c} [\mathcal{G}_c - \delta(t - t')\Delta_i(\mathbf{r}, t)]. \tag{5}$$

(For equation (5) we did not specialize to steady flow; for steady flow the tropopause is stationary so that the masks Δ_i and Δ_Ω have no time dependence.) The point of equation (5) is that the boundary condition (2) is now replaced with a relaxational forcing in the Ω layer. In the limit $\tau_c \to 0$ the forcing acts instantaneously and the boundary condition (2) is enforced exactly in the Ω layer, and hence on the tropopause, and thus $\lim_{\tau_c \to 0} \mathcal{G}_c = \mathcal{G}$. In the limit $\tau_c \to 0$ the dominant balance of (5) in the Ω layer for $t > t'$ is between the flux divergence and the sink $\rho \mathcal{G}_c / \tau_c$.

[13] Consider now a volume element $d^3r = dAdn$ of the Ω layer, where dA is an area element of the tropopause at point $\mathbf{r}_\Omega$. In the limit $\tau_c \to 0$, the rate with which the mass of (Ω_i, t') air is "lost", or more precisely, has its (Ω_i, t') label removed in d^3r for $t > t'$, is balanced by the inflow of (Ω_i, t') air through dA into the volume element. This is simply a statement of Gauss' law (divergence theorem), which gives the normal component of the advective-diffusive mass flux through dA, denoted by $\mathcal{J}$ as

$$\mathcal{J}(\mathbf{r}_\Omega|\Omega_i, \xi) = \rho(\mathbf{r}_\Omega) dn \lim_{\tau_c \to 0} \mathcal{G}_c(\mathbf{r}_\Omega, |\Omega_i, \xi)/\tau_c. \tag{6}$$

Just like $\mathcal{G}$ is a density with respect to transit time ξ so that the mass fraction of (Ω_i, t') air is given by $\mathcal{G}d\xi$, the quantity $\mathcal{J}$ is a flux-density with respect to ξ so that the normal mass flux across dA of Ω_i air of age (transit time since last Ω_i contact) in the interval (ξ, $\xi + d\xi$) is given by $\mathcal{J}d\xi$. Note that in the $\tau_c \to 0$ limit, $\mathcal{J}d\xi$ is a one-way flux (zero return flux back into the stratosphere) because $\mathcal{G}$ is instantly set to zero in the Ω layer. The quantity $\mathcal{J}$ is thus a flux-density distribution that partitions the one-way flux across the tropopause with respect to transit time ξ (for non-steady flow with respect to source time t'). For steady flow, we have for sufficiently small τ_c that

$$\mathcal{J}(\mathbf{r}_\Omega|\Omega_i, \xi) = dn\, \rho(\mathbf{r}_\Omega) \mathcal{G}_c(\mathbf{r}_\Omega|\Omega_i, \xi)/\tau_c, \tag{7}$$

so that the ξ dependence of $\mathcal{J}$ is simply proportional to that of $\mathcal{G}_c$.

[14] It is worth noting that in the $\tau_c \to 0$ limit the one-way flux density $\mathcal{J}$ becomes singular at $\xi = 0$ for $\mathbf{r}_\Omega \in \Omega_i$, in which case the flux of Ω_i air regardless of transit time $\int_0^\infty \mathcal{J} d\xi$ is in fact infinite [*Hall and Holzer*, 2003; *Primeau and Holzer*, 2006]. However, for $\mathbf{r}_\Omega \in \Omega^{ci}$, or for τ_c arbitrarily small but finite, $\mathcal{J}$ remains integrable with respect to transit time. For convenience, we will assume from here on that τ_c is sufficiently small for (7) to hold but finite, so that $\mathcal{J}$ remains free of singularities until we take the limit $\tau_c \to 0$ at the end.

3.1. Mean Age on Exit, Γ_Ω

[15] At a point $\mathbf{r}_\Omega$ on the tropopause, the one-way flux-density distribution $\mathcal{J}(\mathbf{r}_\Omega|\Omega_i, \xi)$ has the natural interpretation as a distribution of $\Omega_i \to \mathbf{r}_\Omega$ transit times. Indeed, we can define a mean transit time from Ω_i to tropopause point $\mathbf{r}_\Omega$ as

$$\Gamma_\Omega(\mathbf{r}_\Omega, \Omega_i) \equiv \frac{\langle \xi \mathcal{J}(\mathbf{r}_\Omega|\Omega_i, \xi) \rangle}{\langle \mathcal{J}(\mathbf{r}_\Omega|\Omega_i, \xi) \rangle}. \tag{8}$$

Because the Ω_i air exits the stratosphere at $\mathbf{r}_\Omega$ with flux-density $\mathcal{J}$, it is useful to refer to Γ_Ω also as the "mean age on exit". Using (7), we can now express Γ_Ω in terms of $\mathcal{G}$:

$$\Gamma_\Omega(\mathbf{r}_\Omega, \Omega_i) = \lim_{\mathbf{r} \to \mathbf{r}_\Omega} \lim_{\tau_c \to 0} \frac{\langle \xi \mathcal{G}_c / \tau_c \rangle}{\langle \mathcal{G}_c / \tau_c \rangle} = \lim_{\mathbf{r} \to \mathbf{r}_\Omega} \frac{\langle \xi \mathcal{G} \rangle}{\langle \mathcal{G} \rangle} = \lim_{\mathbf{r} \to \mathbf{r}_\Omega} \Gamma(\mathbf{r}, \Omega_i), \tag{9}$$

where we have suppressed the arguments of $\mathcal{G}$ for clarity. Thus, the mean transit time based on the one-way flux density is nothing more than the limit of the ideal mean age $\Gamma(\mathbf{r}, \Omega_i)$ as $\mathbf{r}$ approaches the tropopause. Note that the limit is necessary because on Ω^{ci} the boundary propagator $\mathcal{G} = 0$ so that both the numerator and denominator of (9) are zero. Recasting the limit (9) in terms of the flux of $\mathcal{G}$ as in (8) leads to a non-zero denominator, which is a physical manifestation of L'Hôpital's limit rule.

[16] It is perhaps worth emphasizing that the mean age on exit Γ_Ω should not be confused with the traditional mean age with respect to a single control region (such as the tropical tropopause or the earth's surface) simply evaluated at the tropopause. In other words, if the origin identity of the Ω_i air is only removed (by zeroing the age or "age spectrum") on Ω_i or at the earth's surface, then the corresponding mean age on the tropopause is just the mean time since last Ω_i contact at an interior point that happens to be the tropopause. The mean age Γ_Ω discussed here is the mean age on the tropopause of stratospheric air only – the labeling tracer $\mathcal{G}$ is zeroed on the tropopause, requiring the limits discussed.

3.2. Connection Between $\bar{\tau}$ and Γ_Ω

[17] From the one-way flux-density distribution $\mathcal{J}$ one can immediately infer the mass of the stratosphere whose tropopause-to-tropopause transit time, and hence residence time, will lie in the interval $(\xi, \xi + d\xi)$ because this mass will be flushed out of the stratosphere in time ξ with flux $\mathcal{J}(\mathbf{r}|\Omega_i, \xi)d\xi$. In other words, this mass is given by $\mathcal{J}$ flushing out (Ω_i, ξ) air for a time ξ and is hence proportional to $\xi\mathcal{J}$ for steady flow. More precisely, the mass of the stratosphere that had last contact with Ω_i and that will make next contact with tropopause area element dA at $\mathbf{r}_\Omega$ after a residence time in $(\xi, \xi + d\xi)$ is given by

$$\xi\mathcal{J}(\mathbf{r}_\Omega|\Omega_i, \xi)dA\ d\xi \equiv \mathcal{R}(\xi, \Omega_i, \mathbf{r}_\Omega)dAd\xi. \qquad (10)$$

The distribution $\mathcal{R}$ (defined here as a density with respect to ξ and position $\mathbf{r}_\Omega$ on the tropopause that integrates to the mass of the stratosphere) has been referred to as the transport-mass distribution or the residence-time distribution when normalized to integrate to unity [*Primeau and Holzer*, 2006; *Holzer and Hall*, 2008; *Orbe et al.*, 2012]. Note that equation (10) is simply the statement that the residence time ξ is given by the mass in residence-time band $(\xi, \xi + d\xi)$ divided by its flux, that is, $\xi = \mathcal{R}/\mathcal{J}$. This residence time is an entry-to-exit residence time in the same sense as used by *Wernli and Bourqui* [2002] and *Stohl et al.* [2003] in a Lagrangian context.

[18] The mean residence time of air that had last contact with Ω_i and that will make next contact with dA at $\mathbf{r}_\Omega$ is thus naturally defined by

$$\bar{\tau}(\Omega_i, \mathbf{r}_\Omega) = \frac{\langle \xi\mathcal{R}(\xi, \Omega_i, \mathbf{r}_\Omega)\rangle}{\langle \mathcal{R}(\xi, \Omega_i, \mathbf{r}_\Omega)\rangle}. \qquad (11)$$

The timescale $\bar{\tau}$ is the expected mean residence time of $\Omega_i \rightarrow \mathbf{r}_\Omega$ air in the interior of the stratosphere, while Γ_Ω is the mean age of the Ω_i air that exits at $\mathbf{r}_\Omega$. An equivalent expression for $\bar{\tau}$ can also be derived by averaging the mean time since entry plus the mean time to exit over the entire stratosphere, where the weights of the average are provided by the path-density distribution as outlined in Appendix A.

[19] Following the same logic as in (9) and using the fact that $\mathcal{R} \propto \xi\mathcal{J} \propto \xi\mathcal{G}$, we can express equation (11) as the limit

$$\bar{\tau}(\Omega_i, \mathbf{r}_\Omega) = \lim_{\mathbf{r}\rightarrow\mathbf{r}_\Omega} \frac{\langle \xi^2\mathcal{G}\rangle}{\langle \xi\mathcal{G}\rangle}. \qquad (12)$$

It is useful to re-express (12) in terms of the usual centered second moment, or width, Δ of the transit-time distribution. The width of $\mathcal{G}$ at a given point is defined by $2\Delta^2 \equiv \left\langle (\xi - \Gamma)^2\mathcal{G}\right\rangle / \langle\mathcal{G}\rangle$ so that we can rewrite (12) as

$$\bar{\tau} = \lim_{\mathbf{r}\rightarrow\mathbf{r}_\Omega} \left(\Gamma + 2\frac{\Delta^2}{\Gamma}\right) = \Gamma_\Omega\left[1 + 2\lim_{\mathbf{r}\rightarrow\mathbf{r}_\Omega}\left(\frac{\Delta}{\Gamma}\right)^2\right]. \qquad (13)$$

Equation (13) has immediate, interesting implications:

[20] 1. If the flow is purely bulk advective along a single path, then $\Delta = 0$ so that $\bar{\tau} = \Gamma_\Omega$. Thus, for purely bulk-advective flow there is no need to distinguish between the mean residence time and the mean transit time (mean age) at exit. As soon as the transport has multiple paths and/or any eddy-diffusive component, $\Delta > 0$ and the mean residence time will exceed the mean age on exit.

[21] 2. Because the moments of $\mathcal{G}$ can be obtained directly from the mixing ratios of transient tracers [e.g., *Boering et al.*, 1996; *Harnisch et al.*, 1996; *Hall and Waugh*, 1997; *Volk et al.*, 1997; *Strunk et al.*, 2000], the mean residence time $\bar{\tau}$ can be estimated from observational data. In particular, the mean age and width, Γ and Δ, can in principle be obtained from two transient tracers with linearly independent time histories at the tropopause or in the troposphere (e.g., linear and quadratic, or exponential and oscillatory time dependence; see also section 5 below). To the extent that these observationally determined moments can be evaluated near the tropopause and extrapolated to the tropopause in a manner consistent with the boundary conditions (2), estimates of the mean residence time $\bar{\tau}$ are available through equation (13).

[22] To clarify the physical reason for the difference between Γ_Ω and $\bar{\tau}$, and to give an idea of the magnitude of this difference for the stratosphere, we now consider a simple conceptual model and present results from an idealized circulation model. We will return to the issue of observational constraints in section 5.

4. Illustration With Models

4.1. A Conceptual Toy Model

[23] Consider the simplest possible model: Fluid is flowing along a pipe of length L at constant speed v from Ω_i through the interior of the domain to region Ω_f on the boundary of the domain. In this case, the transit-time distribution a distance x from Ω_i is $\mathcal{G}(x|\Omega_i, \xi) = \delta(\xi - x/v)$ for all $x < L$, until x hits Ω_f at $x = L$, where $\mathcal{G}$ is set to zero. Note that because all Ω_i fluid ends up at Ω_f, $\langle\mathcal{G}\rangle = 1$. In this model every fluid element in the pipe takes the same $\Omega_i \rightarrow \Omega_f$ transit time of $\Gamma_1 = L/v$, and the mean age at x is $\langle\xi\mathcal{G}\rangle = x/v$, and the width of the transit-time distribution $\Delta = 0$ everywhere. Thus, the mean age on the tropopause is given by $\lim_{x\rightarrow L} x/v = L/v = \Gamma_1$. And because every particle in the pipe has the same residence time, the mean residence time is $\bar{\tau} = \Gamma_1$ as well, consistent with the general result (13) for $\Delta = 0$.

[24] Now consider two purely advective $\Omega_i \rightarrow \Omega_f$ pipes (Figure 1). For simplicity, we consider both pipes to carry fluid of equal constant density, and to have equal cross sectional area A and flow speed v, so that both pipes have the same flow rate and hence the same flux $J_1 = J_2$ through Ω_f. The fluid has $\Omega_i \rightarrow \Omega_f$ transit time Γ_1 in pipe 1 and Γ_2 in

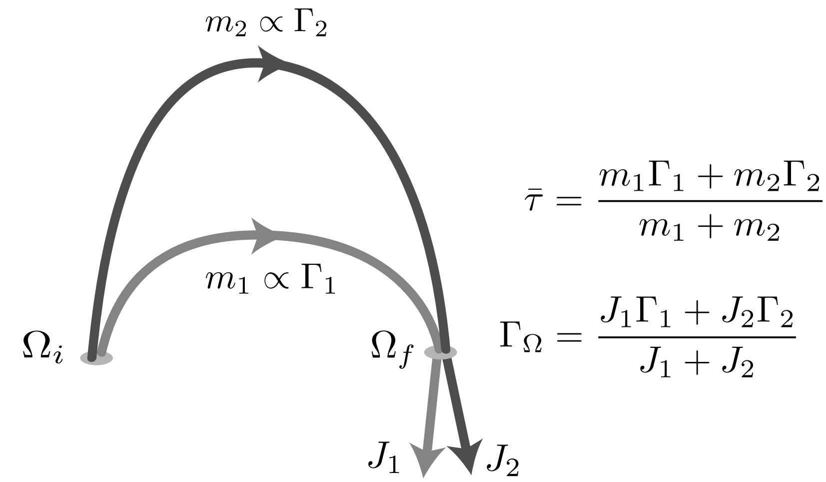

Figure 1. A simple two-pipe model to illustrate the distinction between reservoir mean residence time $\bar{\tau}$ and mean age at exit Γ_Ω: $\bar{\tau}$ is a mass-weighted mean over the entire fluid domain, while Γ_Ω is a mean for which the flow rates provide the weights. If the flow rates J_1 and J_2 in the two pipes (one-way fluxes in the general case) are equal, then mean age at exit on Ω_f is $\Gamma_\Omega = (\Gamma_1 + \Gamma_2)/2$, while the mass-weighted mean residence time of all fluid elements in the pipes is given by $\bar{\tau} = \left(\Gamma_1^2 + \Gamma_2^2\right)/(\Gamma_1 + \Gamma_2) = \Gamma_\Omega + 2\Delta^2/\Gamma_\Omega$, where Δ measures the spread between Γ_1 and Γ_2. For details, see text.

pipe 2. For this system, the mean age on arrival at Ω_f is $\Gamma_\Omega = (J_1\Gamma_1 + J_2\Gamma_2)/(J_1 + J_2) = (\Gamma_1 + \Gamma_2)/2$, reflecting the fact that the same mass arrives at Ω_f per unit time with age Γ_1 from pipe 1, and with age Γ_2 from pipe 2. To compute the mean residence time of all the fluid in pipes 1 and 2, we note that pipe 1 carries a mass $m_1 = \rho A v \Gamma_1$, and pipe 2 carries a mass $m_2 = \rho A v \Gamma_2$. Because each pipe is purely advective the residence time in pipe 1 is Γ_1 and the residence time in pipe 2 is Γ_2, so that the mass-weighted mean residence time is given by $\bar{\tau} = (m_1\Gamma_1 + m_2\Gamma_2)/(m_1 + m_2) = \left(\Gamma_1^2 + \Gamma_2^2\right)/(\Gamma_1 + \Gamma_2) = \Gamma_\Omega + (\Gamma_1 - \Gamma_2)^2/[2(\Gamma_1 + \Gamma_2)]$. Thus, $\bar{\tau} > \Gamma_\Omega$ unless $\Gamma_1 = \Gamma_2$. This result for $\bar{\tau}$ again follows from equation (13) because $\mathcal{G}$ arbitrarily close to Ω_f is given by $\mathcal{G} = [\delta(\xi - \Gamma_1) + \delta(\xi - \Gamma_2)]/2$, so that $\langle \mathcal{G} \rangle = 1$, $\Gamma_\Omega = \langle \xi \mathcal{G} \rangle = (\Gamma_1 + \Gamma_2)/2$, $\langle \xi^2 \mathcal{G} \rangle = \left(\Gamma_1^2 + \Gamma_2^2\right)/2$, and $\Delta = |\Gamma_1 - \Gamma_2|/\sqrt{8}$. It can then be easily be verified that $\bar{\tau} = \langle \xi^2 \mathcal{G} \rangle / \langle \xi \mathcal{G} \rangle = \Gamma_\Omega + 2\Delta^2/\Gamma_\Omega$. (An explicit limit need not be taken here because the purely advective flow in each pipe results in non-zero moments of $\mathcal{G}$ arbitrarily close to Ω_f.)

[25] The difference between mean age on exit Γ_Ω and mean residence time $\bar{\tau}$ stems from the fact that Γ_Ω is a flow rate weighted mean, while $\bar{\tau}$ is a mean for which the mass residing in the interior of the fluid domain provides the weights. For equal flow rates, the pipe with the longer residence time contains more mass because it is flushed more slowly, which skews the average $\bar{\tau}$ toward the longer residence time. The generalization to advective-diffusive flow is provided by equation (10), which states that for steady flow the mass in a given residence-time interval is simply equal to the flow rate of air in that interval acting for a time equal to the residence time.

4.2. An Idealized Atmosphere

[26] To demonstrate the distinction between mean age on the tropopause and mean residence time in a more realistic setting, we present a calculation based on ensemble-mean Green functions and ensemble-mean one-way cross-tropopause flux-density distributions $\mathcal{J}$ computed for an idealized atmosphere. The idealized atmosphere is dry, with zonally symmetric forcings as described by *Polvani and Kushner* [2002] and with a wave number-2 idealized topography in the Northern Hemisphere to provide stratospheric variability [*Gerber and Polvani*, 2009]. Details of the calculation of the model's flux-density distribution $\mathcal{J}$ may be found in the work of *Orbe et al.* [2012].

[27] Figure 2 shows the zonally averaged mean age on the tropopause, Γ_Ω, and the mean residence time, $\bar{\tau}$, for stratospheric air that had last tropopause contact between 8°S and 8°N (Ω_{EQ}). These timescales were computed robustly using the ensemble averaged one-way flux-density distribution $\mathcal{J}$. The latitudinal structure of Γ_Ω and $\bar{\tau}$ is set by the stratospheric circulation, that is, by the combined aggregate action of residual-mean advection and quasi-horizontal eddy-diffusion, as described by *Orbe et al.* [2012]. For this model, Ω_{EQ} air that exits in extra-tropical Northern-Hemisphere latitudes has a mean residence time of ~8.5 years, nearly twice its ideal mean age on exit of ~4.5 years (Figure 2). In other words, the mass-weighted average residence time of Ω_{EQ} air anywhere in the stratosphere, that will eventually exit in the extratropics, is nearly twice as large as the mean age of the Ω_{EQ} air as it exits the stratosphere in the extratropics. Note that from equation (13), the ratio Δ/Γ of Ω_{EQ} air therefore approaches ~0.7 on the extratropical tropopause. The ratio Δ/Γ in the expression for $\bar{\tau}$ may be considered to be a dimensionless measure of the importance of the integrated effect of eddy-diffusion and of the multiplicity of Lagrangian particle paths through the stratosphere. The difference between the timescales Γ_Ω and $\bar{\tau}$ results from the fact that the shorter the residence time, the smaller the mass of the stratosphere that is flushed by the flux of air with that residence time. Thus, the mass-weighted residence time $\bar{\tau}$ preferentially weights long tropopause-to-tropopause transit times compared to the mean age at exit, Γ_Ω.

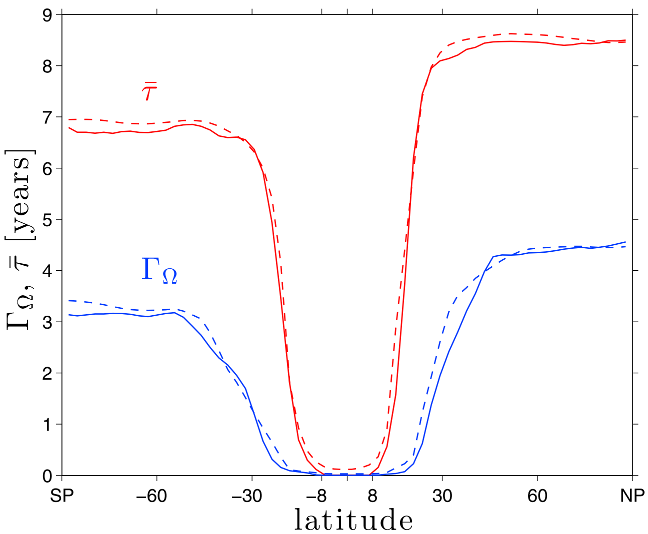

Figure 2. Red lines: the mean residence time $\bar{\tau}$ of air that entered the stratosphere between 8°S and 8°N (Ω_{EQ} air) and that exits at the latitude of the abscissa. Blue lines: the corresponding mean age on exit, Γ_Ω. Solid lines indicate the quantity was computed using the one-way flux-density distribution $\mathcal{J}$, while dashed lines indicate that the limit of moment ratios was used.

[28] Figure 2 also shows as dashed lines Γ_Ω and $\bar{\tau}$ as estimated from the moment ratios of the ensemble-averaged $\mathcal{G}(\mathbf{r}|\Omega_{EQ}, \xi)$ using equations (9) and (12). The limit was taken only approximately by extrapolating the ratio of the moments vertically in pressure coordinates to the zonally averaged tropopause using moments computed from the zonally and ensemble averaged $\mathcal{G}$. A more accurate calculation would take the limit at every point on the tropopause and then average zonally. Nevertheless, the relatively crudely evaluated limits of the moment ratios $\langle \xi^2 \mathcal{G} \rangle / \langle \xi \mathcal{G} \rangle$ and $\langle \xi \mathcal{G} \rangle / \langle \mathcal{G} \rangle$ agree remarkably well with $\bar{\tau}$ and Γ_Ω as computed from $\mathcal{J}$, considering the "zero-over-zero" nature of the limit.

5. Implications

[29] Mean residence time reveals an important aspect of advective-diffusive flow not captured by mean age. While mean age on the tropopause quantifies the residence time of the air that is leaving the stratosphere, the mean residence time quantifies the average time that the air currently in the stratosphere will spend between successive tropopause crossings. The two timescales thus characterize fundamentally different populations of fluid elements: those residing in the interior of the stratosphere for $\bar{\tau}$ and those exiting the stratosphere for Γ_Ω. To emphasize this point it is useful to refer to $\bar{\tau}$ as "reservoir mean residence time" to distinguish it from Γ_Ω, which is the mean residence time of exiting fluid elements. If one is interested in analyzing the budgets of chemical constituents in the stratosphere, rather than the composition of air that is leaving the stratosphere, reservoir mean residence time is an appropriate timescale.

[30] The implications of our results are slightly different depending on whether they are applied in a model context or if they are applied to observations. In the model context, the boundary propagator can be computed explicitly as a passive tracer subject to the required boundary conditions, and the limits of the moment ratios yielding Γ_Ω and $\bar{\tau}$ can be performed by extrapolating the moment ratios to the tropopause, as demonstrated in Figure 2. However, the most robust method for computing these quantities is to compute them from the one-way cross-tropopause flux-density distribution $\mathcal{J}$, which avoids a "zero-over-zero" limit. In addition, $\mathcal{J}$ allows the computation of the mass of the stratosphere that had last contact with specified region Ω_i and that will next contact specified region Ω_f [*Orbe et al.*, 2012]. Although $\mathcal{J}$ can easily be computed with a transport model, $\mathcal{J}$ is unfortunately not directly available from tracer observations.

[31] Observationally based estimates of mean age on exit and reservoir mean residence time carry a different set of considerations. Such estimates are based on the propagator nature of $\mathcal{G}$. Given a tiling of the tropopause with tiles Ω_i and the mixing ratios $\chi(\Omega_i, t')$ of a conservative, passive tracer with known time history for each tile (which can be chosen to be arbitrarily small), the mixing ratio in the interior of the stratosphere is given by

$$\chi(\mathbf{r}, t) = \sum_{\Omega_i} \int_{-\infty}^{t} dt' \mathcal{G}(\mathbf{r}, t|\Omega_i, t') \chi(\Omega_i, t'). \quad (14)$$

From the known time histories $\chi(\Omega_i, t')$ on the tropopause and known interior measurements of the mixing ratio $\chi(\mathbf{r}, t)$, equation (14) can be deconvolved for $\mathcal{G}$ [e.g., *Johnson et al.*, 1999; *Holzer et al.*, 2010]. For a finite number of tracers, this deconvolution is highly underdetermined, but N linearly independent time histories can be used to constrain N integrals of $\mathcal{G}$. For example, it follows immediately from (14) that if the time history at the boundary $\chi(\Omega_i, t')$ is a linear function of t', then the mixing ratio for steady flow is given by the age-lagged boundary value so that $\chi(\mathbf{r}, t) = \sum_i f(\mathbf{r}|\Omega_i) \chi(\Omega_i, t - \Gamma(\mathbf{r}, \Omega_i))$, while a time history that is an nth order polynomial in t' yields a mixing ratio that is a linear combination of the temporal moments $\langle \xi^m \mathcal{G} \rangle$, for $m = 0 \ldots n$. For points $\mathbf{r}$ sufficiently deep in the stratosphere, (14) can in practice be simplified by assuming that only tracer from the tropical tropopause contributes significantly (the sum over Ω_i can be replaced with a single term). With the further approximation of a steady circulation, a linear time history then immediately provides $\langle \xi \mathcal{G} \rangle$ and a quadratic time history provides $\langle \xi^2 \mathcal{G} \rangle$, while other functional forms for $\chi(\Omega_i, t')$ provide combinations of the moments of $\mathcal{G}$, which are typically dominated by only the first few moments [e.g., *Hall and Waugh*, 1997]. However, because we are interested in extrapolating ratios of the moments to the tropopause, the approximation of a dominant single tropical patch will have to be carefully examined for the available tracer data. The details of observational estimates of reservoir mean residence time from extant tracer measurements are beyond the scope of this paper but we believe that analyzing tracer observations for reservoir mean residence time will be fruitful and yield new physically interpretable information on stratospheric transport that is not available from mean-age estimates.

6. Summary and Conclusions

[32] We have defined Γ_Ω, a physically meaningful mean age of stratospheric air on the tropopause, by generalizing the usual mean age since last tropopause contact to the mean age of the fraction of stratospheric air that had last tropopause contact with only a specific subregion Ω_i of the tropopause. In contrast to the traditional stratospheric mean age, which is defined to be zero everywhere on the tropopause, Γ_Ω is a generally nonzero "mean age on exit" that characterizes the mean transit time of stratospheric air that had previous tropopause contact at Ω_i as it exits the stratosphere. Because the fraction of air that had last tropopause contact at Ω_i is naturally defined to be zero elsewhere on the tropopause, the physical basis of Γ_Ω is the one-way flux density $\mathcal{J}$ of (Ω_i, t') air that is induced by holding the (Ω_i, t') label, $\mathcal{G}$, at zero on the tropopause.

[33] The partitioning of the one-way flux density $\mathcal{J}$ according to the entry-to-exit transit time ξ immediately gives the steady-flow result that the mass of air that resides in the stratosphere for a time in the interval (ξ, $\xi + d\xi$) is proportional to $\mathcal{R} \equiv \xi\mathcal{J}$. The usual bulk statement, that the residence time in a reservoir is given by the ratio of resident mass to the flux out of the reservoir [e.g., *Volk et al.*, 1997; *Hall and Waugh*, 2000], is thus generalized to stratospheric air in a specific interval of residence time between successive tropopause crossings, and equation (8) is the corresponding bulk statement $\Gamma_\Omega = \langle\mathcal{R}\rangle/\langle\mathcal{J}\rangle$ for the mean age on exit.

[34] By showing that Γ_Ω and $\bar{\tau}$ are both determined by the one-way cross-tropopause flux density of (Ω_i, t') air, we were able to elucidate the physical difference of these timescales and to develop a simple expression for $\bar{\tau}$ in terms of the moment ratios of the transit-time distribution in the limit as the field point is moved to the tropopause. Expressing $\bar{\tau}$ as a moment ratio allows $\bar{\tau}$ to be evaluated in terms of the mean and width of the transit-time distribution extrapolated to the tropopause, quantities that can be estimated from the observed mixing ratio of two or more transient tracers with independent, known time histories on the tropopause.

[35] Reservoir mean residence time $\bar{\tau}$ and mean age on exit Γ_Ω are only equal for purely advective flow along a single path, but for general advective-diffusive flow $\bar{\tau} > \Gamma_\Omega$ because these timescales evaluate the mean tropopause-to-tropopause transit time over different populations of fluid elements: The mean age on exit Γ_Ω is an average based on the one-way flux-density distribution $\mathcal{J}(\mathbf{r}_\Omega|\Omega_i,\tau)$, which bins fluid elements as they are exiting the tropopause according to their place of origin Ω_i and their residence time τ. Reservoir mean residence time $\bar{\tau}$ is an average based on the transport-mass distribution $\mathcal{R}(\tau,\Omega_i,\mathbf{r}_\Omega) \propto \tau\mathcal{J}(\mathbf{r}_\Omega|\Omega_i,\tau)$, which bins all the fluid elements that are in the interior of the stratosphere at any given time according to Ω_i, exit location $\mathbf{r}_\Omega$, and τ.

[36] The timescales $\bar{\tau}$ and Γ_Ω give independent information on the tropopause-to-tropopause transport through the stratosphere. While $\Gamma_\Omega(\mathbf{r}_\Omega, \Omega_i)$ is simply the mean age of the air from tropopause region Ω_i exiting the stratosphere at location $\mathbf{r}_\Omega$, $\bar{\tau}(\Omega_i,\mathbf{r}_\Omega)$ is the expected mean residence time of Ω_i air that is in the interior of the stratosphere and will eventually exit at $\mathbf{r}_\Omega$. Unlike Γ_Ω, the reservoir mean residence time $\bar{\tau}$ depends additionally on the second moment of the transit-time distribution, which can be regarded as quantifying the aggregated action of eddy-diffusive mixing. While the fundamental underlying flux-density distribution can readily be computed using any circulation model capable of carrying conserved tracers, we hope that the results derived here will stimulate observationally based estimates of reservoir mean residence time.

Appendix A: Residence Time as a Domain Averaged Sum of Entry and Exit Times

[37] While the boundary propagator $\mathcal{G}(\mathbf{r}|\Omega_i,\xi_\uparrow)$ gives the distribution of times $\xi_\uparrow$ since last contact with Ω_i, the equivalent quantity for the time-reversed adjoint flow $\widetilde{\mathcal{G}}(\mathbf{r}|\Omega_f,\xi_\downarrow)$ gives the distribution of times $\xi_\downarrow$ for air at $\mathbf{r}$ to recontact the tropopause at Ω_f [*Holzer and Hall*, 2000]. The mean $\Omega_i \rightarrow \Omega_f$ transit time of air in a parcel at $\mathbf{r}$ is given by [*Holzer and Primeau*, 2008]

$$\tau(\mathbf{r},\Omega_i,\Omega_f) = \frac{\langle\xi_\uparrow\mathcal{G}\rangle}{\langle\mathcal{G}\rangle} + \frac{\langle\xi_\downarrow\widetilde{\mathcal{G}}\rangle}{\langle\widetilde{\mathcal{G}}\rangle}. \tag{A1}$$

The reservoir mean residence time $\bar{\tau}$ is then obtained by averaging over the entire mass of the stratosphere

$$\bar{\tau}(\Omega_i,\Omega_f) = \frac{\int_S \bar{\eta}(\mathbf{r},\Omega_i,\Omega_f)\tau(\mathbf{r},\Omega_i,\Omega_f)d^3r}{\int_S \bar{\eta}(\mathbf{r},\Omega_i,\Omega_f)d^3r}, \tag{A2}$$

where the integrals are performed over the entire stratosphere and $\bar{\eta}(\mathbf{r},\Omega_i,\Omega_f)$ is the density per unit volume of the $\Omega_i \rightarrow \Omega_f$ paths, regardless of residence time. In terms of the full path density, η, which is additionally distributed according to residence time [*Holzer and Primeau*, 2006, 2008; *Holzer*, 2009a, 2009b], we have $\bar{\eta} \equiv \int_0^\infty \eta(\mathbf{r},\tau,\Omega_i,\Omega_f)d\tau$.

[38] The formulation (A2) of the reservoir mean residence time in terms of the path density is equivalent to equation (11) [*Holzer and Primeau*, 2008], except that transport to a finite region Ω_f is considered, which can be taken to be arbitrarily small. (For exit through a finite Ω_f, $\mathcal{R}(\xi,\Omega_i,\mathbf{r}_\Omega)$ is simply integrated over Ω_f before it is used in (11).) The point of (A2) is that it makes explicit that the reservoir mean residence time $\bar{\tau}$ takes into account the entire stratosphere, while mean age only considers the fluid at field point $\mathbf{r}$.

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References

Björkström, A. (1978), A note on the relation between average age and average transit time in natural reservoirs, *Tellus*, *30*, 185–188.

Boering, K. A., S. C. Wofsy, B. C. Daube, H. R. Schneider, M. Loewenstein, and J. R. Podolske (1996), Stratospheric transport rates and mean age distribution derived from observations of atmospheric CO_2 and N_2O, *Science*, *274*, 1340–1343.

Bolin, B., and H. Rhode (1973), A note on the concepts of age distribution and transit time in natural reservoirs, *Tellus*, *25*, 58–62.

Eyring, V., et al. (2006), Assessment of temperature, trace species, and ozone in chemistry-climate model simulations of the recent past, *J. Geophys. Res.*, *111*, D22308, doi:10.1029/2006JD007327.
Gerber, E., and L. Polvani (2009), Stratosphere-troposphere coupling in a relatively simple AGCM: The importance of stratospheric variability, *J. Clim.*, *22*, 1920–1933.
Haine, T. W. N., H. Zhang, D. Waugh, and M. Holzer (2008), On transit-time distributions in unsteady circulation models, *Ocean Modell.*, *21*, 35–45.
Hall, T. M., and M. Holzer (2003), Advective-diffusive mass flux and implications for stratosphere-troposphere exchange, *Geophys. Res. Lett.*, *30*(5), 1222, doi:10.1029/2002GL016419.
Hall, T. M., and R. A. Plumb (1994), Age as a diagnostic of stratospheric transport, *J. Geophys. Res.*, *99*, 1059–1070.
Hall, T. M., and D. W. Waugh (1997), Timescales for the stratospheric circulation derived from tracers, *J. Geophys. Res.*, *102*, 8991–9001.
Hall, T. M., and D. W. Waugh (2000), Stratospheric residence time and its relationship to mean age, *J. Geophys. Res.*, *105*, 6773–6782.
Hall, T. M., D. W. Waugh, K. A. Boering, and R. A. Plumb (1999), Evaluation of transport in stratospheric models, *J. Geophys. Res.*, *104*, 18,815–18,839.
Harnisch, J., R. Borchers, P. Fabian, and M. Maiss (1996), Tropospheric trends for CF_4 and C_2F_6 since 1982 derived from SF_6 dated stratospheric air, *Geophys. Res. Lett.*, *23*, 1099–1102.
Holzer, M. (2009a), The path density of interhemispheric surface-to-surface transport. Part I: Development of the diagnostic and illustration with an analytic model, *J. Atmos. Sci.*, *66*, 2159–2171.
Holzer, M. (2009b), The path density of interhemispheric surface-to-surface transport. Part II: Transport through the troposphere and stratosphere diagnosed from NCEP data, *J. Atmos. Sci.*, *66*, 2172–2189.
Holzer, M., and T. M. Hall (2000), Transit-time and tracer-age distributions in geophysical flows, *J. Atmos. Sci.*, *57*, 3539–3558.
Holzer, M., and T. M. Hall (2008), Tropospheric transport climate partitioned by surface origin and transit time, *J. Geophys. Res.*, *113*, D08104, doi:10.1029/2007JD009115.
Holzer, M., and F. W. Primeau (2006), The diffusive ocean conveyor, *Geophys. Res. Lett.*, *33*, L14618, doi:10.1029/2006GL026232.
Holzer, M., and F. W. Primeau (2008), The path-density distribution of oceanic surface-to-surface transport, *J. Geophys. Res.*, *113*, C01018, doi:10.1029/2006JC003976.
Holzer, M., I. G. McKendry, and D. A. Jaffe (2003), Springtime trans-Pacific atmospheric transport from East Asia: A transit-time probability density function approach, *J. Geophys. Res.*, *108*(D22), 4708, doi:10.1029/2003JD003558.
Holzer, M., F. W. Primeau, W. M. Smethie Jr., and S. Khatiwala (2010), Where and how long ago was water in the western North Atlantic ventilated? Maximum entropy inversions of bottle data from WOCE line A20, *J. Geophys. Res.*, *115*, C07005, doi:10.1029/2009JC005750.
Johnson, D. G., K. W. Jucks, W. A. Traub, K. V. Chance, G. C. Toon, J. M. Russell III, and M. P. McCormick (1999), Stratospheric age spectra derived from observations of water vapor and methane, *J. Geophys. Res.*, *104*, 21,595–21,602.
Neu, J. L., and R. A. Plumb (1999), Age of air in a “leaky pipe” model of stratospheric transport, *J. Geophys. Res.*, *104*, 19,243–19,255.
Nir, A., and S. Lewis (1975), On tracer theory in geophysical systems in the steady and non-steady state. Part I, *Tellus*, *27*, 372–383.
Orbe, C., M. Holzer, and L. M. Polvani (2012), Flux distributions as robust diagnostics of stratosphere-troposphere exchange, *J. Geophys. Res.*, *117*, D01302, doi:10.1029/2011JD016455.
Plumb, R. A. (2002), Stratospheric transport, *J. Meteorol. Soc. Jpn.*, *80*, 793–809.
Polvani, L. M., and P. J. Kushner (2002), Tropospheric response to stratospheric perturbations in a relatively simple general circulation model, *Geophys. Res. Lett.*, *29*(7), 1114, doi:10.1029/2001GL014284.
Primeau, F. W. (2005), Characterizing transport between the surface mixed layer and the ocean interior with a forward and adjoint global ocean transport model, *J. Phys. Oceanogr.*, *35*, 545–564.
Primeau, F. W., and M. Holzer (2006), The ocean’s memory of the atmosphere: Residence-time and ventilation-rate distributions of water masses, *J. Phys. Oceanogr.*, *36*, 1439–1456.
Stohl, A., H. Wernli, P. James, M. Bourqui, C. Forster, M. A. Liniger, P. Seibert, and M. Sprenger (2003), A new perspective of stratosphere-troposphere exchange, *Bull. Am. Meteorol. Soc.*, *84*, 1565–1573.
Strunk, M., A. Engel, H. Schmidt, C. M. Volk, T. Wetter, I. Levin, and H. Glatzel-Matteier (2000), CO_2 and SF_6 as stratospheric age tracers: Consistency and the effect of mesospheric SF_6-loss, *Geophys. Res. Lett.*, *25*, 341–344.
Volk, C. M., J. W. Elkins, D. W. Fahey, G. S. Dutton, J. M. Gilligan, M. Loewenstein, J. R. Podolski, K. R. Chan, and M. R. Gunson (1997), Evaluation of source gas lifetimes from stratospheric observations, *J. Geophys. Res.*, *102*, 25,543–25,564.
Waugh, D., and T. Hall (2002), Age of stratospheric air: Theory, observations, and models, *Rev. Geophys.*, *40*(4), 1010, doi:10.1029/2000RG000101.
Wernli, H., and M. Bourqui (2002), A Lagrangian “1-year climatology” of (deep) cross-tropopause exchange in the extratropical Northern Hemisphere, *J. Geophys. Res.*, *107*(D2), 4021, doi:10.1029/2001JD000812.