Mathematics and Ethics: The Two Sciences with Demonstrable Truths

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Intro

This talk has just two main ideas.

The first one is that mathematics and ethics are very alike. And they're *un*like the range of "ordinary" sciences from physics to ornithology to sociology. Mathematics and ethics are about eternal, necessary truths accessible to pure reason. Other sciences require observation, measurement and experiment for reason to work on. You have get out in the wet to find the facts in the empirical sciences. Not so in mathematics or ethics.

That was an idea familiar to Plato. He advised that future rulers of the state should undergo extensive training in mathematics to fit their minds for the appreciation of the Good. The idea has gone off the boil somewhat since then. So let's see why we should revive it. Let's start with 5 minutes on mathematics. (I say "5 minutes" so that any mathphobics in the audience have a time horizon to look forward to. But my intention is that you follow what I say. The mathematical examples are simple, but revealing.)

Maths

Mathematics is not fundamentally about formulas and rules.

If your memory of mathematics at school *is* all formulas and rules, that's unfortunate.

Nor is mathematics about what to do. Plato makes fun of those who think mathematics is about what to do because they hear mathematicians talking about actions like adding and subtracting, extracting square roots and extending lines.

True, mathematics does include some formulas, rules and actions. But they're peripheral matters of technique. They're not what mathematics is *really* about. Here's an example of what mathematics *is* really about, a classic from the eighteenth century.

Pic/voiceover

The citizens of Königsberg, in north-eastern Europe, often walked over the bridges of their city, as in the diagram (top and bottom are the banks of the river, there are two islands in the middle, and seven bridges connecting the four land areas, as shown.) [pause] The citizens found that it was very difficult to walk over all the bridges once, without walking over at least one of them twice. [pause] You try it: we'll pause for a few seconds while see if you can find (in your mind's eye) a path over all the bridges, that doesn't go over some bridge twice."

Pause, Telemann music?

The great mathematician Leonhard Euler studied the problem. He *proved* that the citizens' hunch was right. It is *impossible* to walk over all the bridges just once. That is an impossibility - a proved impossibility - not about some abstract model or idealization, but about the real arrangement of bridges in a real city.

Now *That* is mathematics!

Here's a second example, a simpler one.

Pic/voiceover

Consider two rows of three stars, one above the other. That's $2 \times 3 = 6$ stars.

Now consider them as three columns of two. That's 3×2. It's the same lot of stars, just divided into parts differently. So we see that $2 \times 3 = 3 \times 2$. Indeed, we see not only that 2×3 is in fact 3×2, but that 2×3 *must* be 3×2. That's a necessary truth, and one which applies to the very physical stars in front of us.

A last example, for you to do in your mind's eye, or what used to be called the imagination until poets took over that term. Imagine a wooden cube painted all over. Now divide it in three equally in each direction (horizontally, and in both vertical directions). First, how many little cubes are there?

3x3x3 = 27. Here's picture of a Rubik's cube. I'll take the picture away, but please keep it in your mind.

Now here's the more difficult question. How many of those 27 little cubes are painted on exactly two sides?

OK, I'll give you the picture back to help you.

It's 12. One in the middle of each edge (4 on top, 4 down the sides, 4 on the bottom). That's a necessary truth that you can come to know by visualizing.

What we learn from mathematical examples like these is that there exists a world of necessities, understandable by pure thought, which constrain what happens the real world – the physical, non-abstract world. That opens our minds to becoming attuned to the necessities of ethical reality, as we'll see shortly.

Exponential graphs: time spent looking at exponential graphs meme

Plato says:

"What we have to consider is whether the greater and more advanced part of [mathematics] tends to facilitate the apprehension of the idea of good. That tendency, we affirm, is to be found in all studies that force the soul to turn its vision round to the region where dwells the most blessed part of reality, which it is imperative that it should behold." (*Republic* 527e)

Plato says too what exactly it is about studying mathematics that helps with ethics. It's the act of *understanding* in mathematics that leads the mind to contemplate the purely intelligible realities that stand behind appearances.

Ethics

Next, ethics

Ethics is not fundamentally about formulas and rules.

If your memory of ethics at school or church *is* all formulas and rules, that's unfortunate.

Nor is it fundamentally about what to do (though some of it is about that). That's a controversial thing to say, certainly. I'll explain.

It's not unusual to take ethics to be *by definition* about what you ought to do. Peter Singer writes, for example, "Ethics is about how we ought to live. What makes an action the right, rather than the wrong, thing to do? What should our goals be? ..."

Unlike some of his other ethical views, that one is mainstream.

So traditionally, ethics has not often been understood as the study of eternal truths about an independently existing objective subject-matter in the way mathematics has been. It has been usual to see ethics as about human actions – their rightness, their obligatoriness, what is a duty and what is permitted, which actions are virtuous. Even "realist" ethical philosophies that take it to be an objective matter which actions are right, such as Kantian universalisability or natural law or classical utilitarianism, suppose that it is fundamentally actions (or action-oriented virtues) that exhibit moral qualities such as rightness.

Pic, voicover

We need to stand back from those particular theories and get the big picture of ethics …

Some of you may be fans of the P.G. Wodehouse stories. They're about the young aristocrat Bertie Wooster, who's not the sharpest tool in the shed, and his smooth butler Jeeves. In each story, Bertie gets into a scrape which Jeeves has to extract him from. In the very first story, where Bertie and Jeeves meet, the problem is that Bertie has got engaged to a young lady who is totally unsuitable. This is Florence in the original illustration.

Her unsuitability is revealed in the passage where Bertie says:

"she was particularly keen on boosting me up a bit nearer her own plane of intellect. She was a girl with a wonderful profile, but steeped to the gills in serious purpose. I can't give you a better idea of the way things stood than by telling you that the book she'd given me to read was called *'Types of Ethical Theory'"* [Jeeves takes charge]

Now *Types of Ethical Theory* is not an invention of Wodehouse. It's a real book, written by James Martineau and published in 1885. And not a bad book too, though I wouldn't recommend its prose style. It says that there are essentially two types of ethical theory. I adapt his theory somewhat, but not too much.

"Ethics may pursue their course and construct their body of doctrine either from the moral sentiments outwards into the system of the world; or from the system of the world inwards to

the moral sentiments. The former method may be called the *Psychologic* …" (1885/2011, 3-4)

The first type of ethical theory – spoiler alert, the wrong type – Martineau calls "psychologic" and we might now call naturalist.

It thinks of ethics as primarily evolved and learned patterns of thinking and behaviour which serve some societal purpose. It often begins with some Darwinian fairytale about how primitive societies needed to develop altruistic behaviour so there was enough cooperation to survive. Then it's supposed that sentiments of approval and disapproval train people in how to follow the customs of the tribe, ensuring tribal cohesion. In Annette Baier's phrase, it thinks of ethics as "traffic rules for self-assertors" (Annette Baier, 1985). Ethics is really prudence, when it isn't just etiquette.

I wonder could you have a 'psychologic' or naturalist theory of mathematics? You might say that arithmetic behaviour must have evolved too. We need to do accountancy, since some of us owe the king more sheep than others, and we make up rules to help us do that (or maybe the king makes them up). There is actually a book called *A Social Constructivist Philosophy of Mathematics*, but it's an uphill battle - Because you understand that accountancy practice must follow the pre-existing laws of arithmetic, not the other way round. Arithmetic is just how it is, necessarily, with numbers and additions. So it's normative for the actions of accountancy, but it is itself neither essentially normative nor about actions. Nor could it have evolved differently – some tribes have developed more mathematics than others, but not mathematics that contradicts other tribes' mathematics.

The *other* kind of ethical theory says that there's an external standard that evolved behaviours must be held to. Not every evolved behaviour has the rosy glow of altruism. Rape and genocide, for example. Genghis Khan is estimated to have some 16 million living descendants, so rape and genocide certainly worked for him, evolutionarily speaking. But according to a non-naturalist view of ethics, that does nothing to make those behaviours right.

Now, any properly objectivist ethics, such as Kantian or natural law theories, says there's an external standard that personal or tribal behaviours ought to conform to. But the purest form of objectivist ethics compares it much more closely with mathematics, as a pure science of necessities.

Here we come to the second main idea of the talk. I said at the beginning that the first main idea was that mathematics and ethics were alike. Now we come to the second idea, which is needed to appreciate the first.

That idea is that ethics is not *fundamentally* about what to do.

There are two basic reasons for thinking that. Firstly, what we are most disturbed by ethically – what most violently forces itself on us as ethically objective – is not anything to do with actions, but the terribleness of suffering. What makes something a *tragedy* is first and foremost the happening of serious evil to some being of worth.

Secondly, whenever we ask why some action is right or wrong, we find we are led back to reasons that are not themselves about action but concern the good or evil of those affected. For example, what makes the act of killing wrong is in the first place the evil of the death of the victim (rather than the action's violating some rule or being contrary to some virtue). That

explains why the rule against killing can be relaxed in the case of killing in self-defence, since then there is a conflict between the evils of the death of the victim and the death of the attacker.

So what is ethics most fundamentally about, if not actions? I'll come back to that, but first, things will be clearer if I take an interlude on casuistry.

Casuistry

I am sure that some people are feeling uncomfortable about comparing ethics with a very abstract science like mathematics. Perhaps you are thinking like this: there are child geniuses in mathematics and music, but there aren't in literature and ethics. In ethics, you need long experience of life and careful reflection to reach wisdom and prudence. Otherwise you'll take a seminarian-deductive approach to moral problems and try to deduce answers from fixed principles. Your advice will be worthless because you don't understand life.

You'll all nod agreement to that, especially if you're old. I agree with it too. But, solving moral problems is not ethics. It's casuistry, which is the solving of particular moral cases, especially cases where there are conflicting considerations. Or rather, casuistry *is* ethics, but only one corner of it. Didn't I just tell you that working out what to do is not the central thing in ethics?

Now, I love casuistry. The Catholic Church has a long history in it, going back to manuals for confessors in the early middle ages. It once virtually owned the field. 70 or 80 years ago, you could write in to the *Australasian Catholic Record* and Fr Nevin would answer your tricky questions.

Is it permitted to deliberately confess to a deaf priest?
Of course not. That defeats one of the purposes of confession.

A harder one: Is an excommunicate obliged to attend mass? [pause] The answer is complex. That was a more relevant case than you'd think. Ben Chifley while Prime Minister used to go to mass but slip out before communion, because he was automatically excommunicated for marrying in a Presbyterian church.

When the Sixties came and free thought reigned, casuistry was ditched as a dog-eared part of the old order.

But no sooner had casuistry gone out the window, than it had to be reinvented as applied ethics. Business ethics and bioethics saved many a philosopher from unemployment. At first there were complaints that philosophers were dealing with unlikely imaginary cases. Then scientists started mixing human sperm with rat ova and frozen embryos inherited fortunes when their parents died in a plane crash. You can't beat real life for throwing up implausible scenarios.

Now we're very concerned about health care allocation. If there's a shortage of ICU beds, should I as an old person with a shorter expected lifespan give my place to a young person, the way Maximilian Kolbe in Auschwitz took the place of a married man with children? [pause] Difficult questions.

That's all important, very important. But it's not core ethics.

Casuistry stands to ethics as accountancy stands to number theory. Sure, to do complex accountancy and value companies it's not enough to be expert in number theory. That's true, but it's a distracting platitude.

Ethics too involves a comparison between what is, which can be seen "by the eye", and what ought to be, which can be understood only by the mind. If a distribution of resources differs from a just division, the actual distribution can be measured but the just one is a standard recognisable only by thought. If an act of killing was in self-defence, that fact can be established without moral input, but whether it was justified requires reference to an outside ethical standard which can only be accessed by the mind. Thus training in the necessities of mathematics has at least the potential to induce an appreciation of the difference between observational facts and abstract standards which can be appreciated only by the understanding.

Equality

There are two things left to do. One is to tell you about equality in mathematics and ethics, and the other is to explain what *is* the foundational concept in ethics – what it is that ethics is most fundamentally about.

These are connected. Let's start with equality.

Whether you see mathematics and ethics as alike or not, you'll be aware that both have a lot to say about equality.

As everyone knows, mathematics is full of equals signs and equations. Perhaps too many for some tastes.

Let's go back to the example of the stars.

The point was that the six stars were equal, identical, to one another as stars. Maybe at the physical microlevel they are not identical, but they're equally "a visible star on the screen". That's why the two rows of three are equal to each other, and the three columns of two are equal to one another. That's what creates the equality, 2 x 3 = 3 x 2.

I choose a second and last example of equality in mathematics because it's both one that everybody has some idea of, and is also a little like how equality works in ethics. It's about probabilities of dice. If you throw two dice and look at the two faces that come up, the total on them can be anything from 2 to 12: two ones give you 2 while 2 sixes give you 12. Here is a graph of the probabilities of getting each of the possibilities from 2 to 12. Take a moment to look it over. [pause] Read especially the numbers along the bottom, which are the possible totals

You can see that the totals in the middle, around 7, are more probable – they come out more often – than the totals at the end, around 2 and 12. That calls for explanation. Why is that?

The reason is that it's a consequence of *equality* – the equality of individual pair outcomes, like 3 on the first die and 4 on the second. Here's a table of the 36 possibilities, each of which has an equal chance of occurring. Take a moment to look at them, and to mentally add the two numbers in a few of the boxes.

You can see now why equality of the basic 36 outcomes creates *in*equality in the chances of different totals. Some totals can be made up of basic outcomes in few ways, some in many ways.

Pascal, who discovered these things in the seventeenth century, realized that he'd come across something surprising: that by use of equality, the variability of chance outcomes could be reduced to formula. He wrote: "by thus uniting the demonstrations of mathematics to the uncertainty of chance, and reconciling what seem contraries, it can take its name from both sides, and rightly claim the astonishing title: the Geometry of chance."

Now back to ethics. Possibly the most influential book in political ethics in the last 50 years is Rawls' *Theory of Justice*.

His distinctive thesis is that adequate principles of justice are those which would be chosen by rational agents concerned to maximize their share of such things as "rights and liberties, opportunities and powers, income and wealth" and "a sense of one's own worth". They are to design the just system without knowing what position they will hold in it, thus behind a "veil of ignorance" which is fair to everyone by putting them in the same initial position. His proposals were reasonable, like prioritizing the worst off, and the ability of his simple axioms to generate them proved very popular. Everyone loves a model, especially one with deductions from principles of equality.

Rawls says, echoing Pascal on probability, "We should strive for a kind of moral geometry, "with all the rigor which this name connotes." He does admit we're not quite there yet.

Appeals to the equality of persons are very powerful in ethics and politics. Equality before the law is a settled and deep principle, and the High Court of Australia in its Mabo decision gave that as the reason for overturning the doctrine of *terra nullius*. When it comes to healthcare allocation in a crisis, there's a strong commitment to equal rights for everyone. The system tries very hard not to depart from that. And so on.

Well, equality of what? It would be no good having persons equal, if we were all equally worthless, like cockroaches. In that case there'd be no point in the moral enterprise, such as devising systems to make sure everyone gets a fair go.

Now I'm in a position to tell you what the foundational concept in ethics is. It's the worth of persons, and in particular the equal worth of persons.

That's what explains why the death of a human is a tragedy while the explosion of a lifeless galaxy is just a firework. It explains what makes dilemmas dilemmas – typically, different people's rights conflict. It explains why we are so strongly committed to equal rights and shocked by violations of that, like slavery.

If you are wondering whether the principle of the equal worth of persons will solve all moral. dilemmas, the answer is no. That would be like expecting Euclid's axioms of geometry to tell you how far it is to the shops. We're not doing casuistry here, but foundations of ethics. What it explains is why each side of a dilemma has moral weight (for example in a case like self-defence, where the worth of the victim and the attacker come into conflict).

Obviously we need some story about what it is about humans that gives them moral worth. That's for another time.

The problem of evil

Leibniz and best of all possible worlds referring to Konigsberg?

Peroration

W.D. Ross wrote in 1930:

> That an act, *qua* fulfilling a promise, or *qua* effecting a just distribution of good, or *qua* returning services rendered … is *prima facie* right … is self-evident just like a mathematical axiom, or the validity of a form of inference, is evident. The moral order expressed in these propositions is just as much part of the fundamental nature of the universe … as is the spatial or numerical structure expressed in the axioms of geometry or arithmetic.
>
> In our confidence that these propositions are true there is involved the same trust in our reason that is involved in our confidence in mathematics; and we should have no justification for trusting it in the latter sphere and distrusting it in the former. In both cases we are dealing with propositions that cannot be proved, but that just as certainly need no proof.

Since then, many relativist and historicist currents have undermined that kind of robust belief in the absolute objectivity of ethical truths. They've insinuated that moral rules are all made up by evolution or society or the powerful and hence we shouldn't believe them. Those currents of thought have had less success with undermining belief in mathematical truths. Well, no success. The mathematical structure of the universe *is* part of its fundamental and necessary nature and the truths describing it are knowable with certainty. The parallels between mathematics and ethics suggest that the same can be said of the ethical structure of reality. That revives the essentials of Plato's view that mathematics is the ideal body of knowledge for preparing the soul to understand ethical truths.