Quantifying Risk: Traditions and practices in medieval Western Christian world

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i. Definitions and Terminology

Late medieval Italy developed a very complex and numerate commercial culture. Weights and measures, computations with arabic numerals, careful recording of dates and times, credit instruments in banking and the development of algebra led to a level of numeracy not seen before. The nature of commerce is to be very concerned about the future, and some of these quantitative techniques especially concerned future quantities and the management of risks of future events – insurance and life annuities, tables of compound interest, and (to a much lesser extent) ideas on the outcomes of dice games and lotteries.

It proved difficult to develop a language for risk (for example to explain what is bought by an insurance premium). Medieval (and ancient) thought lacked any idea of either stochastics or statistical inference. On a modern understanding, there are three aspects of probability: stochastics (such as the individual unpredictability but long-run average stability of dice throws); statistical inference from data (such as inferring the efficacy of drugs from frequencies of cures in experiments); and uncertain conclusions from evidence (as in "proof beyond reasonable doubt" in law or historical evidence). The last of these was familiar in medieval thought, especially in legal contexts, with a range of terminology in Latin such as *suspicio, coniectura, presumptio, indicatio, probabilis, verisimilis* ("likely"), all of which have meanings close to their descendants in modern European languages. The first two aspects of probability were almost absent.

However, a language was developed for discussing uncertain quantities involved in commerce, such as insurance rates, prices of life annuities, and estimates of forward prices for crops. They involved intuitive estimates of probabilities, based on experience. The basic concepts for discussing these probabilities had been developed by ancient Roman law, which allowed the pricing of a "hope" (*spes*) or "peril" (*periculum*) as an entity distinct from a thing; for example one could purchase a future catch of fish and "The contract is valid even if nothing results, because it is the purchase of an expectancy (*spei*)." Analysis of such cases by medieval legal

theorists led to words such as hope, peril, risk (*risicum*, Italian *rischio*) and probability (*probabilitas*) being applied to the quantity of risk that could be assigned a price in cases such as insurance and annuity contracts.

These quantities were not however based, as might be done now, on tables of collected statistics such as the mortality tables now used by life insurers. They were based on the experience of experts, but intuitively and so as to take into account the multitude of variable relevant factors, such as reports of pirates in the sea lanes that would increase the rates for maritime insurance.

ii. Written Sources and Artifacts

Most medieval thought on risk is connected with commercial practice in one way or another, whether directly or in the course of legal and mathematical reflection on practice.

1. Contracts of merchants

The primary business documents involving risk are contracts. Many maritime insurance contracts survive for the insurance of an individual ship on a particular voyage, mainly Italian from the fourteenth and fifteenth centuries. They state a premium but it is not usually possible to see on what basis it was calculated.

Business documents are generally not very explicit about the concepts involved, such as risk. It is also true that business history is a specialized and not widely practised variety of historiography, so much of the primary material has not yet been intensively studied.

2. Guides to merchant practice

A few guides to business practice survive, such as Pegolotti's *Practica della mercatura* of about 1340, which includes a table of compound interest.

3. Mathematical works based on merchant arithmetic

Luca Pacioli's *Summa de Arithmetica* (1494) is the main printed work summarizing the techniques of the Italian merchant schools developed in the previous three centuries.

4. Tables of compound interest

Pegolotti's guide contains a table of compound interest and Pacioli's treatise assumes an ability to calculate compound interest, but the tables were regarded as commercial secrets and no more examples survive from the period.

5. Treatises on canon and civil law and moral theology

Works on canon law are much more informative about the concepts involved in commercial practice. Usury, the lending of money at interest, was forbidden by the church law of the middle ages. Merchants and bankers found that inconvenient and their practice often evaded the letter of the law. But it did not do so with impunity, given the power of the church. The prohibition on usury led to difficult problems concerning the relation of interest to risk, necessitating complex legal analysis. For example, if a loan is made with an extra charge to cover the risk of default, is that charge interest or is it really a price for risk (which would be legitimate)? Many complex contracts involving risk were entered into, some of which included genuine charges for risk and some of which were fictions designed to evade the prohibition on usury. Subtle and generally reasonable analyses of the probabilities in contracts like options, insurance and life annuities are found in such legal-moral treatises in Latin as Peter John Olivi's *On Sale, Purchase, Usury and Restitution* (of about the 1290s), Alexander Lombard's *Treatise on Usury* (1307), Lorenzo de Ridolfi's *On Usury* (1403), Johannes Nider's *On the Contracts of Merchants* (about 1400) and Pedro de Santarém's *On Insurance and Merchants' Bets* (1488).

6. Books on games

Rules of how to play various simple and complex games with dice are available in books like Alfonso X's *Book of Games,* but generally without any probabilistic analysis. As we will see, a few manuscripts show a very minimal such analysis.

7. Commentaries on poetry

A few Italian commentaries on poetry (of pseudo-Ovid and Dante) contain some simple but successful analyses of dice outcomes.

8. Oresme's treatises against astrology

Nicole Oresme, in his attacks on astrology, advanced some powerful probabilistic analysis to show that astrological predictions were not credible.

9. Artifacts

Non-written sources are unhelpful. Many medieval dice and some manuscript images of dice games survive, but give little information about how dice were used or how players thought about the games. The existence of biassed dice, it is true, suggests a considerable knowledge of how to cheat. They include ones with too many high or low faces (presumably to be secretly substituted for fair ones) and ones that look fair but are weighted by inserted deposits of

mercury.

iii. Techniques and Manifestations

Commerce is an inherently uncertain enterprise. It involves taking calculated risks about the future. Some ships founder, some return laden with goods. Crops fail or thrive. Business ventures and investments prosper or not, driven by a multitude of unpredictable factors. By modern standards, the risks undertaken by medieval merchants were very high.

Businessmen could pray, consult oracles, and practise divination as well as anyone else. But in addition it came to be recognised, especially in late medieval Europe, that risks could – to a limited extent and in some circumstances – be evaluated and managed by rational methods. The future income from a fixed-interest investment could be calculated by a table of compound interest. The risk of ships failing to return could be roughly estimated and an insurance premium paid to cover the loss. Grain prices varied, but it was generally predictable that they would be low after the harvest and high in spring, because of the relative abundance of grain. The sale of life annuities for an initial lump sum might in an individual case be profitable (if the annuitant died soon) or not, but the predictability of average lifespan allowed them to be priced so as to make an overall profit.

These methods differ from prognostication as normally understood in several ways. Divination and similar methods aim to predict a singular event (and perhaps allow it to be avoided). Risk methods such as insurance do not predict individual events but make statements only about what can be expected to happen in the mass or on average. In that sense they are less ambitious than divination. But in another way they are more ambitious, since by dealing with the mass of cases they permit risk management, that is, a way of securing the future that protects against whatever may happen. The premium paid for an insurance protects the insured against financial loss, if the disaster insured against occurs.

iv. Development of Techniques and Concepts

a. Mercantile Techniques

Tables of compound interest

For predicting the future, tables of compound interest are of special interest because they

display the future values with exactitude and certainty – even more so than astronomical tables. They can do so because what they predict is a human artifact – the money that a bank agrees to pay in the future for money deposited with it.

A depositor places money in a bank and the bank agrees to pay a fixed percentage of the deposit each year, and also to add the interest to the deposit and pay interest on that as well. So the money amount paid in interest increases each year, as the deposit grows. The calculations are complex and painful, especially with the arithmetical methods available in the middle ages (which used fractions but not decimals). So tables of values were very useful.

The Florentine merchant Francesco Balducci Pegolotti includes a table of compound interest in his *Pratica della Mercatura*, a large handbook of information useful in international trade of about 1340 which contains also tables of weights and currencies and descriptions of the route to Cathay. It tabulates the interest on 100 lire, for interest rates from 1% to 8% in steps of half a percent, for up to 20 years.

Pegolotti's table is the only known medieval table of compound interest. It is preserved in a single manuscript of 1472. Tables were not printed until 1558. The reason for the rarity of tables is presumed to be the prohibition on usury, and the fact that tables were regarded as industrial secrets. However mathematical problems on compound interest are found in various mathematical works in the fifteenth century, which assume an ability to calculate interest. Luca Pacioli's *Summa de Arithmetica* (1494) mentions the rule that to determine the number of years for an investment at compound interest to double, one should divide the interest rate into 72. (For example, an investment at 2% compound interest takes about 36 years to double.)

Actually, the certainty of compound interest tables in predicting the future is not perfect. It is possible that the bank offering to pay the interest may default. That was by no means a trivial consideration in view of the spectacular bank failures of the Middle Ages. The huge Bardi firm for which Pegolotti worked collapsed in 1347 and he was engaged in dealing with the consequences of the bankruptcy.

Limiting risks by forward prices, compensation and bankruptcy

Forward contracts provided a way of stabilising future finances. In a typical example, an English monastery had established itself as a reliable supplier of wool. The large Italian trading firms handled the export of wool to continental destinations. The firm could offer to buy the

monastery's wool clip several years ahead – sometimes up to twenty years – for a fixed price payable now. The monastery gained ready cash in the present, the buying firm gained a discount on the price plus a guaranteed supply, any implicit interest payment on the money advanced was invisible, and both sides were protected against the vagaries of future price movements. Both sides could thus plan their operations with a more secure view of the future.

A forward contract provides for certain goods to be delivered at a fixed date in the future, for a price agreed on now. The price thus depends on an estimate of what will be a fair price at the future time. In a case like grain, prices varied to a degree predictably over the year, as grain was more plentiful and so cheaper after harvest than after winter. That provided an opportunity to cover up interest payments and evade the usury prohibition. The distinguished canon law commentator Hostiensis, writing in the mid-thirteenth century, explained clearly how fair prices ought to vary with expectations about the future, and how those expectations are based on what normally happened:

> What therefore if one gives a price at harvest time, with the amount [of grain] to be delivered at Easter? It is clear from the intention of the words that he is to be considered usurious. It is the common and accustomed course in all regions, and common opinion, which are to be taken notice of and expected (*Digest* 43.12.1.1; 33.7.18.2; *Decretals* 3.28.9). And if you say to me, that sometimes the contrary happens [i.e., the price does not rise between harvest and Easter], I reply, that the laws are not adapted to those things which happen rarely (*Digest* 1.3.5); but rather, to those which happen for the most part, and frequently (*Novels* 94.2). (Franklin 2001, 164)

Similar concepts apply to the valuation of future earnings. If someone has had their goods requisitioned by the state, or has lost limbs through a work accident, they may deserve compensation for loss of future earnings. How is that to be measured? It must involve an estimate of what would have happened in the normal or probable course of things. Peter John Olivi, writing around the 1290s, says that is not the general custom to require compensation in the case of lost limbs, because the cause is too remote from the effect, but if it is,

> If you say that he who lost limbs is now deprived of the mechanisms and occupations from which he could derive profit, then it should be said that either the depriver is required to restore only as much as the probability of profit weighs (*quantum ponderat probabilitas talis lucris*), or that … common custom excuses him from any restitution. Perhaps this custom is because often such a probability for the whole life of one who has lost limbs is not at all uncertain, but is of such weight or price as could scarcely be

> compensated for, at least in popular estimation. I believe though that a rich man who has cut off someone's limbs is bound to support him, if he needs support. (Olivi 1980, 91; Franklin 2001, 266)

In traditional societies, commercial risk-taking is discouraged because of the fearsome consequences of bankruptcy, such as debt slavery or imprisonment. One of the points of the prohibition on usury was that it prevented the spiral of debt leading to such consequences. In modern law, the future for an insolvent debtor is not so severe, because bankruptcy law allows the bankrupt to make a "clean slate" free of debt, so as to be able to start again. That was an idea of medieval Roman and canon law, based on some hints from ancient Roman law. It contained the downside of financial risks and allowed merchants and bankers to accept the risks necessarily attendant on the highly variable business environment.

Although individual future events cannot be predicted, there is something about them in the mass which gives rise to a hope (expectation, *risicum*) that can be quantified and given a price.

Life annuities

The essential idea of life annuities is that although the time of death of a healthy individual is unpredictable, the times of deaths of such individuals in the mass, or on average, are rather predictable. That allows money to be made by offering a contract of a life annuity, that is, an annual payment for life in return for a fixed upfront fee (or sometimes provision of services for life such as food, wood, or health care, called a corrody). Approximate knowledge of average life expectancy enables the product to be priced properly. For the buyer, it provides a very predictable income stream for the future, especially for periods when illness or decrepitude may make earning money impossible.

The possibility of selling the prospect of something in the future for hard cash in the present was tempting, for states and other powerful bodies that could offer the purchaser reliability over a lifespan, such as monasteries. In the thirteenth and fourteenth centuries, a trade in both perpetual and life annuities grew up. The cities of Germany commonly raised money through the sale of annuities from the thirteenth century. Hamburg standardly offered 10% of the purchase price per year on life annuities, as against 6.66% for perpetual annuities (that is, annuities inheritable by heirs); Douai in 1324-5 offered 10% for life annuities, 5% for perpetual ones. The difference between life and perpetual represents some kind of implicit quantification of the expected life span. Ghent in the later fourteenth century gave 10% for a life annuity,

6⅔% for an annuity on two lives (one payable until both purchasers had died), and 5% for perpetual annuities, although these rates varied widely. In the fourteenth century, there is found the first discrimination on grounds of age, to reflect the different expectancies of life at different ages; Nordhausen in 1350 offered 1 mark per year for a down payment of 10 for persons between 40 and 50, but 1 mark per year for 8 to those between 50 and 60. In Artois in 1399, the rate of interest on a life annuity offered to a man of 58 was twice that offered to a child.

Theoretical analysis again came from canon law commentators. Alexander Lombard writes, in his *Treatise on Usury* (1307), that it might seem that life annuities are unjust contacts, since sometimes twenty-five years olds are seen to live a long time and gain back many times the initial price they paid; if the price is such that they would regain it in eight years, "although they may live less than those eight years, it is more probable (*probabilius*) that they will live twice that." However, he argues, such contracts are not necessarily unjust. It is simply a matter of finding an equitable price by balancing the probabilities:

> Such equality can be saved, when life annuities are sold. This is when the price is of such quantity that, after weighing with care and consideration the age of the buyer and his health, and the risks concerning the profits from the possessions, it does not appear that either the buyer or the seller has notably the better side. If such equity is destroyed, it is certain the contract cannot be made, and is not licit … the contract is licit, because the risk and doubt falls on both sides; for the sale is for a time for which it is doubtful whether he will survive more or less, and the uncertainty of the time makes equality on both sides. (Alexander Lombard 1962, 154, 156; Franklin 2001, 270-1)

Insurance

The idea of insurance was invented by Italian merchants around 1350. The owner of a ship about to go on a voyage pays another person a premium; if the ship returns safely, the contract lapses, while if it the ship is lost the insurer pays the owner an agreed large sum, in compensation for the loss. As the name "insurance" implies, the process makes the future more predictable for the venturer, since (for a fee) he ensures that whatever happens, he does not sustain serious loss. The process encourages precision in stating what risks exactly are covered. (Scheller 2017)

The size of the premium represents a quantification of the risk of the voyage, as estimated by the parties to the contract. Rates vary with available information such as the seasons and reports

of wars or pirates. But by the late fourteenth century normal rates for commonly travelled routes had become established, such as 8 percent for Cadiz to Southampton and 4 percent for the Port of Pisa to Naples or Tunis. Such rates indicate an intuitive understanding of how the frequency of future events reflects the frequency of similar past events. That is not explicitly stated, but without some approximate calibration of future to past frequencies, the insurance business will not make money.

A contract written in Crete in 1389 is valuable for showing in detail what the information is on which doubt is based and how it issues in a price. The contract is called an insurance but is close to a pure bet:

> This day 14 October 1389, 13th indiction, as overleaf, I, Niccolino de Fieschi, a Genoese residing at present in Candia, with my heirs, do hereby make known to you Ser Bernardo de Mezzo of Venice, residing in Candia, in your presence and to your satisfaction, and to thine heirs, that as a *griparia*, master Bartolomeo Acardo, entered the port of Candia today and he having said he saw part of a wreck and a barrel on the sea, and suspecting therefore that the ship of Antonio de Barba was wrecked, which sailed a short while ago from here on a voyage to Romania [Constantinople], I insure you for 200 ducats in gold, of good and fair weight, for your part of the cargo existing on said ship; whereby, if the said ship of Antonio de Barba was wrecked, counting from the time she left this port and up to the whole of the present day, understanding that certainty is had of this, I am bound to and must give and pay to you 200 ducats in gold of good and fair weight, here in Candia, safe on land. And for this insurance, according to our agreement, I have received from you 25 gold ducats.
>
> Nevertheless, should anything new happen in Candia, during the whole of this day, whereby we shall know for certain that the said ship of Antonio de Barba has sunk, then in that case I shall not be bound to you for the present insurance and this paper must be held of no value and must be null and void, because I issue this insurance to you for the doubt had and that one still has and not for any certainty. (Stefani 1958, 76, 207; Franklin 2001, 275)

Insurance expanded beyond maritime risks. Insurance on life is found from 1399 (but only for short terms), and from the early fifteenth century there are contracts covering the risk of death in pregnancy, of both wives and slaves. In such areas, the risk can be expected to be rather constant and independent of special knowledge, unlike shipping risks.

The difference between insurance and betting was not clearly understood, and indeed the distinction is a fine one. In the fifteenth century merchants are found making "insurances" on the lives of the Pope and the Doge of Venice, on horses, and on the duration of a conclave. Those practices had to be forbidden, considering the temptation to bring about the event bet on. An intermediate case is insurance against the occurrence of a plague, available at Genoa for 4 percent for a year.

While merchants' documents did not explain the nature of these contracts, legal commentators well-informed about commercial practice did. The nature of insurance is explained very clearly by the Florentine jurist Lorenzo de Ridolfi in 1403, and cleared of any suspicions that insurance is a cover for usury:

> Of insurances (*securitatibus*) … when you send your merchandise by sea or land to certain parts, and I take upon myself the risk, and agree that for every 100 of value of that merchandise, you pay me a certain quantity of money … something is given for something done. It is not given as a loan, since no loan is involved, but something is received for assuring the merchant of his goods, which are at risk by sea or land. Besides, no stake is involved, so it is false to say that something is received over and above the stake; there is no building without a foundation … (Franklin 2001, 276-7)

The first book on insurance, *On Insurance and Merchants' Bets*, by the Portuguese jurist Pedro de Santarém (Santerna), was published in 1552 though written in 1488. Santerna's conception of the contract is expressed in the simple formula, "I undertake the peril, for your giving me money." While admitting that insurance and bets can be a cloak for usury, he argues that generally a good intention in the parties must be presumed. For given the general uncertainty of life, everyone must bet about the future ("since the Pope does not know the secrets of the heart, and cannot divine them, he judges from likelihoods, and common accompaniments, and so is sometimes mistaken.") There are various cases where it is important to determine legally whether the price of an insurance is just. Thus, if someone charges 16 percent in circumstances where 8 percent is usual, it may be inferred that the insurance was intended to cover a second voyage, and if a ship's master defers sailing until a more dangerous time of the year, a price previously just may become unjust. Reinsurance is permitted, that is, insuring an insurance against the default of the insurer. There is a complicated discussion of the difference between "normal" unusual events like storms, and very rare ones, "that occur once in a thousand years".

Santerna asks, can the rule that a contract of sale is invalid if the price exceeds the just price by more than half apply to an insurance? Firstly, it needs to be determined whether an insurance can be said to have a definite value at all:

> It can be said that the insurer sells only the hope of a future outcome, of which there can well exist a sale … from the fact that this hope is uncertain, it might not seem capable of estimation such that in respect of it there could be said to be exceeding of half the just price of its value. But, this is not to be estimated at how much the thing or goods would be worth in case the peril was realised, but at how much the doubtful event should likely (*verisimiliter*) be estimated. In which case the price seems to be constituted with respect to that hope. (Santerna 1971, 336; Franklin 2001, 277)

A few similar discussions can be found at the same time in Jewish law, which also forbade lending at interest and had to explain why the acceptance of risk in marine insurance was legitimate.

The late fifteenth century saw the invention of a subtle but morally dubious use of insurance in an effort to render usury legitimate and thus secure investments against loss. It was agreed that both partnerships and insurance contracts were legitimate. So an investor could invest a sum in a venture, then insure that against both default and against the return being less than some fixed amount, say 5%. Those three contracts were legitimate. If however they were all made with the same person, the effect was the same as lending him money at 5% fixed interest. The moral theorist Angelo de Clavasio defended the "triple contract" in 1485, as did many later writers. Despite complaints from moralists of more rigorous views, it heralded the end of the prohibition on usury.

b. Mathematics of games

Dice

Dice were used frequently in the middle ages, both for prognostication and for gaming. The literature about them consists more of condemnation of the evils of gambling than of any analysis of probabilities. In principle, dice throws have the same kind of predictability as life annuities: dice throws, like the times of human deaths, are unpredictable individually but have a certain predictability in the mass or on average. That was not understood before modern times.

But the mid-thirteenth century poem *De Vetula*, which purports to be Ovid's account of his life, contains a passage about throws of three dice that understands the question well and correctly calculates how rarely or frequently the different possible outcomes (the possible totals on the three dice) occur (in our language, the probabilities). It rightly distinguishes between partitions (*punctaturae*) and "fallings" (*cadentiae*), and correctly counts both for three dice. Thus the outcome 3 has one partition and one falling, as all three dice must come up 1, whereas 4 has one partition (it can only be made up by two 1's and a 2) but three fallings – 1,1,2 and 1,2,1 and 2,1,1. It is correctly explains that how frequently an outcome occurs depends on how many fallings make it up, so that the outcome 4 occurs three times as often as 3. Thus outcomes,

> … are not, however,
> Of equal value, since the larger and the smaller of them
> Come rarely and the middle ones frequently,
> And the rest, the closer they are to the middle ones,
> The better they are and more frequently they come. (Bellhouse 2000)

Despite *De Vetula* existing in nearly sixty manuscripts and several printed editions, its discoveries on dice seem to have made little impact. A commentary on Dante, possibly from the late fourteenth century, addresses the same problem in similar language, but is more confused about the crucial difference between partitions and "fallings". It does repeat the general idea that "a number which can appear in more ways must occur more frequently." (David 1962, 35; Franklin 2001, 293) The next similar work is a fragment on dice by Galileo, which solves the same problem in much the same way.

Out of this context, with mathematical thought about dice barely in existence, there suddenly appears the statement, and the exact solution, of a difficult problem on the correct division of stakes in an interrupted game of chance. The problem is one of those solved in 1654 by Fermat and Pascal, which constitute their claim to have founded mathematical probability. The trivial cases of an interrupted game are when each of two players needs the same number of points to win. In that case, the stake should obviously be divided equally. The simplest non-trivial case is when one player needs two points to win and the other one. The answer as to how the stakes should be fairly divided if the game is interrupted at that point is not easy, as Pascal and Fermat discovered. The next simplest case is when one player needs three points and the other one. This is the problem correctly solved in an anonymous Italian mathematical manuscript of about 1400. It is done with a complex symmetry argument.

The general problem of the just division of the stake in an interrupted game was mentioned in Luca Pacioli's widely read *Summa de Arithmetica,* printed in 1494 (and solved incorrectly). In this and later discussions of the problem, including that of Pascal and Fermat, the problem is considered a moral-legal one about what is deserved and there is no suggestion that it is connected with the long-run relative frequency of how dice fall. Thus the connection between the mathematics of dice and what can be expected in future dice throws is in this period extremely tenuous.

Lotteries

Lotteries bear an unusual relation to predictions of the future. For the lottery owner, they are a sure bet, being designed to pay out less than the incomings whatever happens (provided, however, that all the tickets can be sold). Like life annuities, they provide a way of making money for the offerer by aggregating individual risks into something predictable overall, the offerer relying on their credibility to pay out substantial sums in the future. For the individual ticketholders, they increase unpredictability, as the buyer trades a sure cost for the mere possibility – indeed, a remote possibility – of a win. They appeal therefore to risk appetite, which is the opposite of the appeal to risk aversion in, for example, insurance.

Lotteries were offered by cities in Burgundy and Italy from the 1440s. They were instituted for civic purposes, sometimes to keep the city solvent, and so appealed to a sense of civic virtue as well as to greed. A lost stake could be regarded as a donation to a worthy purpose. In 1468, Raffaele de' Neri offered to pay 2000 lire to the Milanese treasury for the right to conduct a lottery of 10000 one-lira tickets, with 268 prizes totalling 8000 lire. That would make money only if some prizes were not drawn or were unclaimed.

c. Mathematics of planetary conjunctions

Oresme on the inability of astrology to predict

Nicole Oresme, the mathematical genius of the mid-fourteenth century, advanced a subtle probabilistic argument intended to show that astrology cannot make reliable predictions.

He begins with a simple example of the connection between probability, in the sense of what one ought to believe on evidence, and relative frequency:

> If there were some number as to which it were completely unknown what it is or how great it is, and whether it is large or small — as perhaps the number of all the hours that will pass before Antichrist — it will be likely that such a number would not be a cube number. It is similar in games where, if one should inquire whether a hidden number is a cube or not, it is safer to reply that it is not, since that seems more probable and likely (*probabilius et verisimilius).* (Oresme 1966: 248–51)

The reason is that cubes are rare among numbers: the great majority of numbers are not cubes. The reasoning is thus a form of the statistical (or proportional) syllogism:

> Most A's are B's
>
> This is an A (about which there is no further relevant information)
>
> Therefore this is probably a B

The statistical syllogism is a form of reasoning commonly used intuitively but rarely explicit before the modern theory of statistical testing. Oresme goes on to apply the same reasoning to the more complicated case of commensurability of ratios. Two ratios are said to be commensurable if one is a rational power of the other; for example, the ratios 4/1 and 32/1 are commensurable since $32/1 = (4/1)^{(5/2)}$, but 4/1 and 5/1 are not commensurable. (The idea of fractional powers was itself a discovery of Oresme's.) The vast majority of pairs of ratios are not commensurable, therefore, given an arbitrary set of ratios, it is very likely that none of them are commensurable with any of the others.

The application to astrology is that astrological prediction relies on the recurrence of conjunctions of planets. But if the ratios of times of orbits are incommensurable, there can be no exact recurrence of conjunctions. Since all ratios between the periods of planetary orbits are almost certainly incommensurable, exact recurrence is extremely unlikely and hence, Oresme concludes, astrology can make no predictions.

Readers naturally found that reasoning hard to follow. Oresme's work on the question had virtually no impact and the mathematical ideas in it were lost until modern times.

d. Medieval Classifications and Discussions

The essential contribution of medieval European law was in understanding the nature of "aleatory contracts", those involving chance or risk in itself as a saleable commodity. They included insurance, gambling, speculation and life annuities. Though implicit in ancient Roman law in a few cases, there was no general theory inherited from the ancients. The medieval commentators appreciated that in life annuities, insurance and bets, a price is paid for an uncertainty, which is not a physical thing but an abstract entity – in modern terms, an expectation. Such a quantity is to be estimated by the market, based on what is seen to happen usually.

These conceptual developments concerning risk were not found in Islamic cultures. Authoritative traditions forbade all contracts involving risk or speculation (*gharar*). Thus one could not sell a bird in the air, even one accustomed to return to its nest, nor a ripening crop. So there was no occasion for discussion of risk in contracts as occurred in the Latin West. Nor was there the opportunity for generations of commentary and reflection on the examples and principles of Roman law.

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