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Berkeley on Speed

By James Franklin

Douglas M. Jesseph, *Berkeley's Philosophy of Mathematics.* Chicago: University of Chicago Press, 1993. Pp. xii + 322. US$19.19 PB.

Many philosophers think that they could stride into almost any technical field, bone up on a few details, and show the practitioners a thing or two about their conceptual confusions. One of the few occasions on which something like that actually happened was the publication of Berkeley's *The Analyst*, of 1734, where he attacked the mathematicians' understanding of the foundations of the calculus, as hopelessly confused and contradictory. At issue is the meaning of a derivative, or rate of change of a variable quantity - the 'fluxion' of a 'fluent', in Newton's terminology. If we wish to measure the speed of a moving object, that is, the rate of change of distance, we use a unit like miles per hour. To find the numerical value of an object's speed, therefore, we divide the distance it travels in any time interval by the length of the interval. If the speed is constant, no problems arise: the answer is the same whatever interval is taken: 12 miles divided by 3 hours gives the same answer as 8 miles divided by 2 hours, namely 4 miles per hour. But if the speed is itself variable, conceptual problems arise in trying to explain what the instantaneous speed is, at any given instant. For, the speed calculated from dividing any finite distance traversed by the finite time taken to do so is not an instantaneous speed, but the average speed over the interval. It is natural to approximate the speed at an instant more closely by taking smaller and smaller intervals including that instant, but the problem remains that an instantaneous speed and an average speed are different, both conceptually and numerically. Newton used such doubtfully intelligible language as:

> Fluxions are very nearly as the augments of the fluents generated in equal, but very small, particles of time; and to speak accurately, they are in the *first ratio* of the nascent augments ...

In calculating the speed if the distance travelled in time x is x^n, he first finds the distance travelled in the time between x and $x + o$, divides it by the 'augment' of time o, and finally claims that when the augment o

vanishes, their 'ultimate ratio' is as 1 to nx^{n-1}. Berkeley's criticism is perfectly correct:

> For when it is said, let the increments vanish, or let there be no increments, the former supposition that the increments were something, or that there were increments, is destroyed, and yet a consequence of that supposition, *i.e.*, an expression got by virtue thereof, is retained.

Newton's attempts to speak of the augments as 'nascent' and 'evanescent', and the ratios as 'first' and 'ultimate' attract Berkeley's most famous piece of ridicule:

> And what are these same evanescent increments? They are neither finite quantities, nor quantities infinitely small, nor yet nothing. May we not call them the ghosts of departed quantities?

The mathematicians ignored Berkeley, as one ignorant of the subtleties of their art - until they solved the problem by eliminating infinitesimals in favour of multiple quantification, at which point they said, 'Berkeley was right, which demonstrates the excellence of our new answer'.

Jesseph's treatment of the background, course and immediate aftermath of the *Analyst* controversy occupies the second half of his book. While it is a topic well studied already, and there are no startling revelations, the treatment is complete and judicious, and can be taken as definitive. The first half of the book, on Berkeley's philosophies of geometry and arithmetic, is less charted territory. Berkeley is unusual in that his views on geometry and arithmetic were quite different. This has been common enough since the discovery of non-Euclidean geometry, but was apparently unique earlier. The views themselves are unusual too. Berkeley rejected such Euclidean theorems as the incommensurability of the diagonal and the side of a square, on the grounds that they falsely assumed the infinite divisibility of space; he argued that there was only perceivable extension, and hence no meaning to lengths less than the *minimum visible* (or *minimum tangible*). He proposed to reform geometry along these lines, but never found the time to do so. Jesseph suggests it is impossible to do geometry with a finite number of points — but surely a computer graphics program that calculates how to fill in the pixels on a screen does exactly that? As for arithmetic, Jesseph argues convincingly that Berkeley was the first formalist philosopher of mathematics, in the sense that he believed that arithmetic is about inscriptions of symbols and the rules for calculating with them. The problems with Berkeley's formalism are very clear, confirming the good Bishop's reputation for being excellent target practice for undergraduates, as he makes important mistakes so clearly.

There is a problem with the first chapter, where Jesseph sketches the seventeenth century background. He rightly says that that background is mainly scholastic, and goes so far as to quote a number of scholastics, which is more than most authors who pronounce on the subject do. He makes it clear, too, that the opinions of Wallis, Barrow and, in large measure, Newton, are essentially the same as those of the scholastics. The trouble is that his account of what those writers think is at odds with the plain meaning of the passages he quotes from them. They keep saying things like 'Mathematics is the science of quantity, geometry studying length and other continuous quantities, arithmetic studying discrete quantity'. Quantity is studied 'in abstraction from' other properties things have, such as colour, just as physics studies heat 'in abstraction from' other qualities. Jesseph glosses all such claims as versions of the theory that mathematics studies 'a peculiar kind of mental entities [i.e., abstractions] which can be grasped by the intellect but not found in nature' (p.11). This is quite wrong; the scholastics were realists, and their view is much closer to present-day 'naturalised epistemology' than to the early modern "way of ideas". True, the mistake is not too important for a study of Berkeley, who misread his predecessors in exactly the same way. Still, it does skew the book's treatment of all the thinkers to whom Berkeley is reacting (except Locke).

Jesseph's book is a solid achievement. Nevertheless, from the evidence in it, it seems that there exists, somewhere out in the sphere of possible books, a more exciting one with a title something like *Late Scholastic and Newtonian Philosophy of Mathematics*. Any takers?

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