UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131 Mathematics 1A Algebra S1 2015
TEST 2 VERSION 8b

This sheet must be filled in and stapled to the front of your answers

Student's Family Name Initials Student Number

Tutorial Code Tutor's Name Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working. All answers should be given in the appropriately SIMPLIFIED form.

In any question on solutions to a linear system of equations, you **must** use Gaussian Elimination on an augmented matrix. **NO** marks will be awarded for any other method. Row operations must be specified.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)

   Let $z = -1 - i$ and $w = \sqrt{3} + i$ and $p = \dfrac{z^6}{w^5}$.

   (i) Find $|p|$.

   (ii) Find $\text{Arg}(p)$.

$z = \sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = \sqrt{2}\, e^{-\frac{3\pi}{4}i}$

$w = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 2e^{\frac{\pi}{6}i}$

$|p| = \frac{\sqrt{2}^6}{2^5} = \frac{1}{4}$

$\text{Arg}(p) = -\frac{18\pi}{4} - \frac{5\pi}{6} + 2k\pi = \frac{(3+5)\pi}{6} + 2\pi = \frac{2}{3}\pi$

2. (*3 marks*)

   Use the identity
   $$\sin\theta = \tfrac{1}{2i}(e^{i\theta} - e^{-i\theta})$$
   to write $\sin^6\theta$ in terms of $\cos\theta,\ \cos 2\theta,\ \cos 3\theta, \ldots$.

$\sin^6\theta = \frac{-1}{2^6}(e^{i\theta} - e^{-i\theta})^6$

$= \frac{-1}{64}\left(e^{6\theta i} - 6e^{4\theta i} + 15e^{2\theta i} - 20 + e^{-6\theta i} - 6e^{-4\theta} + 15e^{-2\theta i}\right)$

$= -\frac{1}{32}\left[\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10\right]$

3. (*4 marks*)

   Determine, with reasons, those values of a such that the augmented system
   $$A = \left(\begin{array}{ccc|c} 1 & 1 & 2a & 1 \\ a & 1 & 1 & 2 \\ 2 & 5-3a & a & 5 \end{array}\right)$$
   has i) a unique solution, ii) no solution, iii) infinitely many solutions.

Please write your answers on lined A4 paper and staple to this cover sheet.

$$A \sim \left(\begin{array}{ccc|c} 1 & 1 & 2a & 1 \\ 0 & 1-a & 1-2a^2 & 2-a \\ 0 & 3-3a & -3a & 3 \end{array}\right)\begin{array}{l} \\ R_2 - aR_1 \\ R_3 - 2R_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 2a & 1 \\ 0 & 1-a & 1-2a^2 & 2-a \\ 0 & 0 & 3(2a^2-a-1) & 3(a-1) \end{array}\right)\begin{array}{l} \\ \\ R_3 - 3R_2 \end{array}$$

If $a \neq 1$ and $\underset{\substack{\parallel \\ (2a+1)(a-1)}}{2a^2-a-1} \neq 0$ (i.e., $a \neq 1, a \neq -\frac{1}{2}$) it has a unique solution

For $a = 1$, no solution

For $a = \frac{-1}{2}$ no solution.

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TEST 2 VERSION 8a

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Show all your working. All answers should be given in the appropriately SIMPLIFIED form.

In any question on solutions to a linear system of equations, you **must** use Gaussian Elimination on an augmented matrix. **NO** marks will be awarded for any other method. Row operations must be specified.

QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)

   Let $z = 1 + i$ and $w = 1 - i\sqrt{3}$ and $p = \dfrac{z^6}{w^5}$.

   (i) Find $|p|$.

   (ii) Find $\text{Arg}(p)$.

2. (*3 marks*)

   Use De Moivre's formula to express $\sin(6\theta)$ as a polynomial in powers of $\cos\theta$ and $\sin\theta$.

3. (*4 marks*)

   Determine, with reasons, those values of a such that the augmented system

$$A = \left(\begin{array}{ccc|c} 1 & 2 & -a & 1 \\ 2 & a & 3 & 3 \\ a & 4a-8 & 3a & 5 \end{array}\right)$$

   has i) a unique solution, ii) no solution, iii) infinitely many solutions.

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Q1

$z = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \sqrt{2}\, e^{\pi i/4}$

$w = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 2\, e^{-\frac{\pi}{3}i}$

$|p| = \frac{\sqrt{2}^6}{2^5} = \frac{2^3}{2^5} = \frac{1}{4}$

$\arg(p) = 6 \times \frac{\pi}{4} - 5 \times \frac{-\pi}{3} = \frac{3\pi}{2} + \frac{5\pi}{3} = \frac{19\pi}{6}$

$\text{Arg}(p) = -\frac{5}{6}\pi$

Q2

$\cos 6\theta + i\sin 6\theta = (\cos\theta + i\sin\theta)^6 = \cos^6\theta + 6(\cos^5\theta \sin\theta)i + (\quad) \; 20(\cos^3\theta \sin^3\theta)i + (\quad)$

$+ 6(\cos\theta \sin^5\theta)i + [(\sin\theta)i]^6$.

$\sin 6\theta = 6\cos^5\theta \sin\theta - 20\cos^3\theta \sin^3\theta + 6\cos\theta \sin^5\theta$

Q3

$A \to \left(\begin{array}{ccc|c} 1 & 2 & -a & 1 \\ 0 & a-4 & 3+2a & 1 \\ 0 & 2a-8 & a^2+3a & -a+5 \end{array}\right) \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - aR_1 \end{array} \to \left(\begin{array}{ccc|c} 1 & 2 & -a & 1 \\ 0 & a-4 & 2a+3 & 1 \\ 0 & 0 & a^2-a-6 & -a+3 \end{array}\right)_{R_3 - 2R_2}$

If $a \neq 4$ and $a^2 - a - 6 = (a-3)(a+2) = 0$ or $a \neq 3, -2$, has unique solution

If $a = -2$, no solution.

If $a = 3$, ∞ solution.

For $a = 4$, $A = \left(\begin{array}{cccc} 1 & 2 & -4 & 1 \\ 0 & 0 & 11 & 1 \\ 0 & 0 & 5 & -1 \end{array}\right)$. So no solution

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TEST 2 VERSION 10b

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In any question on solutions to a linear system of equations, you **must** use Gaussian Elimination on an augmented matrix. **NO** marks will be awarded for any other method. Row operations must be specified.

$$z = 2\sqrt{3}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 2\sqrt{3}\, e^{-\frac{5\pi}{6}i}$$

QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)

   $$\operatorname{Arg}(z^{21}) = 21 \times \left(\frac{-5\pi}{6}\right) + 2k\pi = \frac{\pi}{2}, \quad z^{21} = (2\sqrt{3})^{21} i$$

   Let $z = -3 - i\sqrt{3}$. Find the polar form for z and the principal argument and the "$a + ib$" form of z^{21}.

   Powers of real numbers may be left unsimplified.

   line ∥ plane $\iff$ $\lambda\begin{pmatrix}1\\-3\\2\end{pmatrix} + \mu\begin{pmatrix}-3\\6\\-4\end{pmatrix} = \begin{pmatrix}-4\\-3\\2\end{pmatrix}$ has a solution.

2. (*3 marks*)

   Determine whether or not the line

   $$\mathbf{x} = \begin{pmatrix}1\\2\\1\end{pmatrix} + \alpha\begin{pmatrix}-4\\-3\\2\end{pmatrix} \quad \alpha \in \mathbb{R}$$

   $$\left(\begin{array}{cc|c}1 & -3 & -4\\ -3 & 6 & -3\\ 2 & -4 & 2\end{array}\right) \sim \left(\begin{array}{cc|c}1 & -3 & -4\\ 0 & -3 & -15\\ 0 & 2 & 10\end{array}\right) \begin{array}{l} \\ R_2+3R_1\\ R_3-2R_1\end{array}$$

   $$\sim \left(\begin{array}{cc|c}1 & -3 & -4\\ 0 & 1 & 5\\ 0 & 0 & 0\end{array}\right)$$

   is parallel to the plane

   $$\mathbf{x} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\-3\\2\end{pmatrix} + \mu\begin{pmatrix}-3\\6\\-4\end{pmatrix} \quad \lambda, \mu \in \mathbb{R}.$$

   has a solution. So the line is parallel to the plane

   Alternatively. look at their intersection via

   $$\left(\begin{array}{ccc|c}1 & -3 & 4 & 0\\ -3 & 6 & 3 & 1\\ 2 & -4 & -2 & 0\end{array}\right) \sim \left(\begin{array}{ccc|c}1 & -3 & 4 & 0\\ 0 & -3 & 15 & 1\\ 0 & 2 & -10 & 0\end{array}\right)$$

   no solution $\therefore$ parallel.

3. (*4 marks*)

   (i) Find the complex roots of $z^5 + 32 = 0$.

   (ii) Hence factorise $p(z) = z^5 + 32$ into real linear and real irreducible quadratic factors.

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(i) $z^5 = -32 = 2^5 e^{\pi i}$, $\quad z = 2e^{\frac{(2k+1)\pi i}{5}} \quad k = -2, -1, 0, 1, 2$

all roots are $2e^{-\frac{3}{5}\pi i}, 2e^{-\frac{\pi}{5}i}, 2e^{\frac{\pi}{5}i}, 2e^{\frac{3\pi}{5}i}, 2e^{\pi i} = -2$

(ii) $z^5 + 32 = (z+2)(z - 2e^{-\frac{\pi}{5}i})(z - 2e^{\frac{\pi}{5}i})(z - 2e^{-\frac{3}{5}\pi i})(z - 2e^{\frac{3}{5}\pi i})$

$= (z+2)\left(z^2 - 4z\cos\frac{\pi}{5} + 4\right)\left(z^2 - 4z\cos\frac{3}{5}\pi + 4\right).$

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QUESTIONS (*Time allowed: 25 minutes*)

1. (*3 marks*)
   Let $z = -2 - 2i$. Find a polar form of z and the principal argument and "$a + ib$" form of z^{14}.
   Powers of real numbers may be left unsimplified.

$z = 2\sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i\right) = 2\sqrt{2}\, e^{-\frac{3}{4}\pi i}$,

$\operatorname{Arg}(z^{14}) = -\frac{3 \times 14}{4}\pi + 2k\pi \in (-\pi, \pi]$

$= -\frac{1}{2}\pi$

$z^{14} = (2\sqrt{2})^{14}\left(\sin\frac{-\pi}{2} i\right) = -2^{21} i$

2. (*3 marks*)
   Determine if the lines

$$\mathbf{x} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}, \quad \lambda \in \mathbb{R}$$

   and

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}, \quad \mu \in \mathbb{R}$$

   intersect.

We want to see if the equation

$$\begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}$$

has solution.

$$\left(\begin{array}{cc|c} 2 & -2 & 2 \\ 1 & -2 & -1 \\ 4 & -5 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 2 & 4 \\ 0 & 3 & 6 \end{array}\right) \begin{array}{l} R_2 \\ R_1 - 2R_2 \\ R_4 - 4R_2 \end{array} \sim \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right)$$

So the system has solution. Hence, the lines intersect.

3. (*4 marks*)
   (i) Find the complex roots of $z^6 + 27 = 0$.
   (ii) Hence factorise $p(z) = z^6 + 27$ into real linear and real irreducible quadratic factors.

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(i) $z^6 = -27 = 27 e^{\pi i}$ $\quad z = \sqrt{3}\, e^{\frac{(2k+1)\pi}{6} i}$ where $k = -3, -2, -1, 0, 1, 2$

or $z \in \{\sqrt{3} e^{-\frac{5}{6}\pi i}, \sqrt{3} e^{-\frac{1}{2}\pi i}, \sqrt{3} e^{-\frac{1}{6}\pi i}, \sqrt{3} e^{\frac{1}{6}\pi i}, \sqrt{3} e^{\frac{3}{6}\pi i}, \sqrt{3} e^{\frac{5\pi}{6} i}\}$

(ii) $z^6 + 27 = (z^2)^3 + 3^3 = (z^2 + 3)(z^4 - 3z^2 + 3^2)$

$= (z^2 + 3)(z^2 - 2\sqrt{3}(\cos\frac{1}{6}\pi) z + 3)(z^2 - 2\sqrt{3}(\cos\frac{5}{6}\pi) z + 3)$

$= (z^2 + 3)(z^2 - \sqrt{3} z + 3)(z^2 + \sqrt{3} z + 3)$