UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2015
TEST 1 VERSION 8b

This sheet must be filled in and stapled to the front of your answers

Key

Student's Family Name	Initials	Student Number
Tutorial Code	Tutor's Name	Mark

Note: The use of a calculator is NOT permitted in this test

Show all your working. All answers should be given in the appropriately SIMPLIFIED form.

QUESTIONS (*Time allowed: 25 minutes*)

$\underset{\sim}{x} = \begin{pmatrix}5\\4\\-3\end{pmatrix} + \lambda\begin{pmatrix}-3\\-7\\+4\end{pmatrix}$

1. (*2 marks*)

   (i) Find a parametric vector equation of the line in $\mathbb{R}^3$ through the points $A(5,4,-3)$ and $B(2,-3,1)$.

   $5-3\lambda = 17 \Rightarrow \lambda = -4$

   (ii) Is the point $P(17,32,-19)$ on this line? Give reasons.

   $-3+4\lambda = -19$ ✓

   $4-7\lambda = 32$ ✓

   $\boxed{\text{Yes}}$

2. (*2 marks*)

   Find a Cartesian equation for the plane in $\mathbb{R}^3$ which has parametric vector form

   $$\mathbf{x} = \begin{pmatrix}2\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\0\end{pmatrix} + \mu\begin{pmatrix}1\\0\\3\end{pmatrix} \quad \lambda,\mu \in \mathbb{R}$$

   $\begin{pmatrix}2\\1\\0\end{pmatrix} \times \begin{pmatrix}1\\0\\3\end{pmatrix} = \begin{pmatrix}3\\-6\\-1\end{pmatrix}$

   $\left[\underset{\sim}{x} - \begin{pmatrix}2\\1\\2\end{pmatrix}\right]\cdot\begin{pmatrix}3\\-6\\-1\end{pmatrix} = 0$

   $\boxed{3x_1 - 6x_2 - x_3 = -2}$

3. (*3 marks*)

   For the points $A(1,2,6)$, $B(3,4,-2)$, $C(2,-1,3)$ calculate:

   (i) The distance between A and B.

   (ii) $\cos(\angle BAC)$.

   $\text{dist}(A,B) = \sqrt{(3-1)^2 + (4-2)^2 + (-2-6)^2} = \sqrt{72}$

   $= \dfrac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}|\,|\overrightarrow{AC}|} = \dfrac{\begin{pmatrix}2\\2\\-8\end{pmatrix}\cdot\begin{pmatrix}1\\-3\\-3\end{pmatrix}}{\sqrt{72}\sqrt{19}} = \dfrac{20}{6\sqrt{38}} = \dfrac{10}{3\sqrt{38}} = \dfrac{5\sqrt{38}}{57}$

4. (*3 marks*)

   Let ℓ be the line in $\mathbb{R}^3$ through the point $P(3,2,3)$ and parallel to the vector $\mathbf{v} = \begin{pmatrix}2\\1\\2\end{pmatrix}$.

   Let Q be the point with co-ordinates $(2,5,4)$.

   $\overrightarrow{PQ} = \begin{pmatrix}-1\\3\\1\end{pmatrix}$

   (i) Find $\text{proj}_{\mathbf{v}}\left(\overrightarrow{PQ}\right)$. $= \dfrac{\begin{pmatrix}-1\\3\\1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\2\end{pmatrix}}{\|v\|^2}\,v = \dfrac{3}{9}\begin{pmatrix}2\\1\\2\end{pmatrix} = \dfrac{1}{3}\begin{pmatrix}2\\1\\2\end{pmatrix}$

   (ii) Find the co-ordinates $\mathbf{m}$ of the point M on ℓ which is closest to Q.

Please write your answers on lined A4 paper and staple to this cover sheet.

$\underset{\sim}{m} = p + \text{proj}_v(\overrightarrow{PQ}) = \begin{pmatrix}3\\2\\3\end{pmatrix} + \begin{pmatrix}2/3\\1/3\\2/3\end{pmatrix} = \dfrac{1}{3}\begin{pmatrix}11\\7\\11\end{pmatrix}$

SCHOOL OF MATHEMATICS AND STATISTICS
MATH1131/1141 Mathematics 1A Algebra S1 2015
TEST 1 VERSION 8a

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QUESTIONS (*Time allowed: 25 minutes*)

(i) $\underset{\sim}{x} = \begin{pmatrix}3\\5\\-2\end{pmatrix} + \lambda\left[\begin{pmatrix}3\\5\\-2\end{pmatrix} - \begin{pmatrix}7\\-2\\4\end{pmatrix}\right] = \begin{pmatrix}3\\5\\-2\end{pmatrix} + \lambda\begin{pmatrix}-4\\7\\-6\end{pmatrix}$

1. (*2 marks*)

   (i) Find a parametric vector equation of the line in $\mathbb{R}^3$ through the points $A(3, 5, -2)$ and $B(7, -2, 4)$.

   (ii) Is the point $P(19, -23, 20)$ on this line? Give reasons.

   $3 - 4\lambda = 19 \quad \lambda = -4$
   $5 + 7\lambda = -23 \quad \lambda = -4$
   $-2 - 6\lambda = 20 \quad \lambda \neq -4$

   NO.

2. (*2 marks*)

   Find a Cartesian equation for the plane in $\mathbb{R}^3$ which has parametric vector form

$$\mathbf{x} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\0\\-2\end{pmatrix} + \mu\begin{pmatrix}0\\5\\4\end{pmatrix} \qquad \lambda, \mu \in \mathbb{R}$$

   $\underset{\sim}{n} = \begin{pmatrix}3\\0\\-2\end{pmatrix} \times \begin{pmatrix}0\\5\\-4\end{pmatrix} = \begin{pmatrix}10\\-12\\15\end{pmatrix}$

   $\left[\underset{\sim}{x} - \begin{pmatrix}1\\2\\3\end{pmatrix}\right] \cdot \begin{pmatrix}10\\-12\\15\end{pmatrix} = 0$

   or $10x_1 - 12x_2 + 15x_3 = 31$

3. (*3 marks*)

   For the points $A(1, 3, 2)$, $B(4, 4, 1)$, $C(2, 5, 2)$ calculate:

   (i) The distance between A and B.

   $\text{dist}(A, B) = \sqrt{(4-1)^2 + (4-3)^2 + (1-2)^2} = \sqrt{3+1+1} = \sqrt{11}$

   (ii) $\cos(\angle BAC)$.

   $\cos\theta = \dfrac{\vec{AB} \cdot \vec{AC}}{|AB|\,|AC|} = \dfrac{\begin{pmatrix}3\\1\\-1\end{pmatrix} \cdot \begin{pmatrix}1\\2\\0\end{pmatrix}}{\sqrt{11} \cdot \sqrt{5}} = \dfrac{5}{\sqrt{55}}, \quad \theta = \cos^{-1}\left(\dfrac{\sqrt{55}}{11}\right).$

4. (*3 marks*)

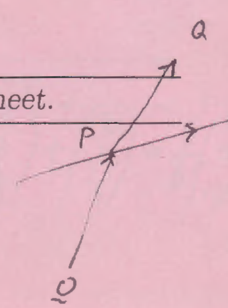
   Let ℓ be the line in $\mathbb{R}^3$ through the point $P\,(1, 2, 3)$ and parallel to the vector $\mathbf{v} = \begin{pmatrix}2\\3\\1\end{pmatrix}$.

   Let Q be the point with co-ordinates $(1, 4, 4)$.

   (i) Find $\text{proj}_{\mathbf{v}}\left(\overrightarrow{PQ}\right)$.

   $\text{Proj}_{\begin{pmatrix}2\\3\\1\end{pmatrix}}\left[\begin{pmatrix}1\\4\\4\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix}\right] = \text{Proj}_{\begin{pmatrix}2\\3\\1\end{pmatrix}}\begin{pmatrix}0\\2\\1\end{pmatrix} = \dfrac{\begin{pmatrix}0\\2\\1\end{pmatrix} \cdot \begin{pmatrix}2\\3\\1\end{pmatrix}}{\begin{pmatrix}2\\3\\1\end{pmatrix} \cdot \begin{pmatrix}2\\3\\1\end{pmatrix}}\begin{pmatrix}2\\3\\1\end{pmatrix} = \dfrac{7}{14}\begin{pmatrix}2\\3\\1\end{pmatrix} = \dfrac{1}{2}\begin{pmatrix}2\\3\\1\end{pmatrix}.$

   (ii) Find the co-ordinates $\mathbf{m}$ of the point M on ℓ which is closest to Q.



Please write your answers on lined A4 paper and staple to this cover sheet.

$m = \underset{\sim}{p} + \text{proj}_{\underset{\sim}{v}}(\vec{PQ}) = \begin{pmatrix}1\\2\\3\end{pmatrix} + \begin{pmatrix}1\\3/2\\1/2\end{pmatrix} = \begin{pmatrix}2\\\frac{7}{2}\\\frac{7}{2}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}4\\7\\7\end{pmatrix}$

UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF MATHEMATICS AND STATISTICS

MATH1131/1141 Mathematics 1A Algebra S1 2015

TEST 1 VERSION 10a

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QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)

   Find a parametric vector equation of the line whose Cartesian equations are

   $$\frac{x_1-4}{3} = \frac{x_3+1}{-2}, \quad x_2 = 6\ .$$

   $\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}4\\6\\-1\end{pmatrix} + \lambda\begin{pmatrix}3\\0\\-2\end{pmatrix}$

   Hence find a point **a** on the line and a vector **v** parallel to the line.

   $=\begin{pmatrix}4\\6\\-1\end{pmatrix}$ $=\begin{pmatrix}3\\0\\-2\end{pmatrix}$

2. (*2 marks*)

   Find the point of intersection of the line ℓ and plane $\mathcal{P}$ with equations

   $$\ell: \quad \mathbf{x} = \begin{pmatrix}1\\2\\4\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\5\end{pmatrix} \quad \lambda \in \mathbb{R}, \qquad \mathcal{P}: \quad x_1 + 4x_2 - 2x_3 = 13\ .$$

   $(1+3\lambda)+4(2+\lambda)-2(4+5\lambda)=13$

   $-3\lambda = 12$ $\quad \lambda=-4$ $\quad$ the intersection $=\begin{pmatrix}-11\\-2\\-16\end{pmatrix}$

3. (*3 marks*)

   For the points $A(1,3,2)$, $B(4,4,1)$, $C(2,5,2)$ calculate:

   (i) The distance between A and B. $\quad \text{dist}(A,B) = \sqrt{(4-1)^2+(4-3)^2+(1-2)^2} = \sqrt{11}$

   (ii) $\cos(\angle BAC)$. $\quad = \theta$

   $\cos\theta = \dfrac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}|\cdot|\vec{AC}|} = \dfrac{\begin{pmatrix}3\\1\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\2\\0\end{pmatrix}}{\sqrt{11}\sqrt{5}} = \dfrac{5}{\sqrt{55}} \qquad \theta = \cos^{-1}\left(\dfrac{\sqrt{55}}{11}\right)$

4. (*3 marks*)

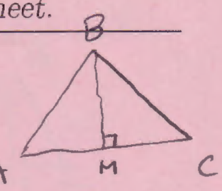
   For the points $A(1,2,3)$, $B(-2,5,3)$, $C(2,4,5)$. $\quad \vec{AB} = \begin{pmatrix}-3\\3\\0\end{pmatrix} = 3\begin{pmatrix}-1\\1\\0\end{pmatrix} \quad \vec{AC} = \begin{pmatrix}1\\2\\2\end{pmatrix}$

   (i) Calculate $\text{proj}_{\overrightarrow{AC}}\left(\overrightarrow{AB}\right)$. $\quad \text{Proj}_{\vec{AC}}(\vec{AB}) = \dfrac{\vec{AB}\cdot\vec{AC}}{\vec{AC}\cdot\vec{AC}}\vec{AC} = \dfrac{3}{1+4+4}\begin{pmatrix}1\\2\\2\end{pmatrix} = \dfrac{1}{3}\begin{pmatrix}1\\2\\2\end{pmatrix}$

   (ii) Calculate the length of the altitude in $\triangle ABC$ through B and perpendicular to AC.

Please write your answers on lined A4 paper and staple to this cover sheet.

$\vec{BM} = \vec{AM} - \vec{AB} = \frac{1}{3}\begin{pmatrix}1\\2\\2\end{pmatrix} - \begin{pmatrix}-3\\3\\0\end{pmatrix} = \frac{1}{3}\begin{pmatrix}10\\-7\\2\end{pmatrix}$ as $\vec{AM} = \text{Proj}_{\vec{AC}}(\vec{AB})$



The altitude $= |\vec{BM}| = \frac{1}{3}\sqrt{100+49+4} = \sqrt{17}$.

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TEST 1 VERSION 10b

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QUESTIONS (*Time allowed: 25 minutes*)

1. (*2 marks*)

   Find a parametric vector equation of the line whose Cartesian equations are

   $$\lambda = \frac{x_1 - 5}{2} = \frac{x_2 - 3}{-4} = \frac{x_3 + 2}{6}. \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$$

   Hence find a point **a** on the line and a vector **v** parallel to the line.

   $\mathbf{a} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$ $\quad \mathbf{v} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$

2. (*2 marks*)

   Find the point of intersection of the line ℓ and plane $\mathcal{P}$ with equations

   $$\ell: \quad \mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} \quad \lambda \in \mathbb{R}, \qquad \mathcal{P}: \quad 2x_1 - x_2 - 2x_3 = 9.$$

   $2(2+4\lambda) - (1+2\lambda) - 2(3+\lambda) = 9$

   $4\lambda = 9+3 = 12 \qquad \lambda = 3 \qquad$ the pt $= \begin{pmatrix} 14 \\ 7 \\ 6 \end{pmatrix}$

3. (*3 marks*)

   For the points $A(1,2,6)$, $B(3,4,-2)$, $C(2,-1,3)$ calculate:

   (i) The distance between A and B. $\quad \text{dist}(A,B) = \sqrt{(3-1)^2 + (4-2)^2 + (-2-6)^2} = \sqrt{4+4+64} = \sqrt{72} = 6\sqrt{2}$

   (ii) $\cos(\angle BAC)$. $= \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}|\,|\overrightarrow{AC}|} = \dfrac{\begin{pmatrix} 2 \\ 2 \\ -8 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \\ -3 \end{pmatrix}}{6\sqrt{2} \cdot \sqrt{19}} = \dfrac{20}{6\sqrt{38}} \qquad \angle BAC = \cos^{-1}\left(\dfrac{10}{3\sqrt{38}}\right)$

4. (*3 marks*)

   For the points $A(1,1,4)$, $B(5,2,1)$, $C(3,3,6)$. $\quad \overrightarrow{AB} = \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix}, \ \overrightarrow{AC} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ choose $\overrightarrow{AC} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

   (i) Calculate $\text{proj}_{\overrightarrow{AC}}\left(\overrightarrow{AB}\right)$. $= \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\overrightarrow{AC} \cdot \overrightarrow{AC}} \overrightarrow{AC} = \dfrac{\begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \dfrac{2}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \overrightarrow{AM}$

   (ii) Calculate the length of the altitude in $\triangle ABC$ through B and perpendicular to AC.

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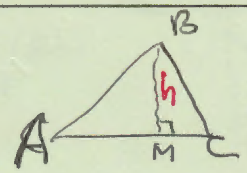
$$\overrightarrow{BM} = \overrightarrow{AM} - \overrightarrow{AB} = \frac{2}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -\frac{10}{3} \\ -\frac{1}{3} \\ \frac{11}{3} \end{pmatrix} = \frac{-1}{3}\begin{pmatrix} 10 \\ 1 \\ -11 \end{pmatrix}$$



$$|\overrightarrow{BM}| = \frac{1}{3}\sqrt{100+1+121} = \frac{1}{3}\sqrt{222}.$$

or $h^2 = |\overrightarrow{AB}|^2 - \left|\frac{2}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right|^2 = 26 - \frac{4}{3} = \frac{74}{3}$