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Finite Dimensional Algebras and Quantum Groups

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Finite Dimensional Algebras and Quantum Groups

*For all their help, encouragement, and infinite patience,
we dedicate this book to our wives and children:*

Wenlian Guo and Zhuoran Deng

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Finite Dimensional Algebras and Quantum Groups

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Abstract. This research/advanced graduate-level text combines, for the first time in book form, the two theories given in its title. As a main feature, the book emphasizes the finite dimensional algebra approach to quantum groups. More precisely, it investigates the Ringel–Hall algebra realization for the positive part of a quantum enveloping algebra associated with a symmetrizable Cartan matrix and looks closely at the Beilinson–Lusztig–MacPherson realization for the entire quantum $\mathfrak{gl}_n$.

The book begins with the two realizations of generalized Cartan matrices, namely, the graph realization and the root datum realization. From there, it develops the representation theory of quivers with automorphisms and the theory of quantum enveloping algebras associated with Kac-Moody Lie algebras. These two independent theories eventually meet, under the umbrella of Ringel–Hall algebras.

Cartan matrices can also be used to define an important class of groups — Coxeter groups — and their associated Hecke algebras. Hecke algebras associated with symmetric groups give rise to an interesting class of quasi-hereditary algebras, the quantum Schur algebras. The structure of these finite dimensional algebras is used to build the entire quantum $\mathfrak{gl}_n$ through a completion process of a limit algebra (the Beilinson–Lusztig–MacPherson algebra).

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