# The 7th Number Theory Down Under
# University of New South Wales, Sydney, Australia

30 September - 3 October, 2019

Room 4082, Level 4

Red Centre Building (East Wing)

WE ARE VERY GRATEFUL TO HAVE RECEIVED SUPPORT FROM:

- The Government of New South Wales
- The Australian Mathematical Sciences Institute
- The Journal of Number Theory
- The School of Mathematics and Statistics, UNSW Sydney

ENJOY THE CONFERENCE!

Alina Ostafe (Chair)
John Roberts
Igor Shparlinski
Liangyi Zhao

# PROGRAM

## Monday 30 September

| | |
|---|---|
| **12:30–14:00** | Registration and Lunch |
| **14:00–14:10** | Introductory Remarks |
| **14:10–15:10** | **Antal Balog**, *Recent results around the Prime Geodesic Theorem*, page 6 [1] |
| **15:10–15:30** | Coffee Break [2] |
| **15:30–15:50** | **Tim Trudgian**, *Mean-square for free for PNT!*, page 16 |
| **15:55–16:15** | **Ethan Lee**, *Chebotarëv's Density Theorem*, page 13 |
| **16:20–16:40** | **Sean Lynch**, *The error term in the prime ideal theorem for noncommutative arithmetic*, page 13 |
| **16:45–16:55** | Break |
| **16:55–17:15** | **Aleksander Simonič**, *Explicit Selberg's zero density estimate*, page 15 |
| **17:20–17:40** | **Matteo Bordignon**, *Explicit bounds on exceptional zeroes of Dirichlet L-functions II*, page 9 |
| **17:45–18:05** | **Takashi Nakamura**, *Symmetric Tornheim double zeta functions*, page 14 |
| **18:10–20:00** | Reception (Red Centre, 3rd Floor, Staff Common Room) |

[1] The page number following a talk title is where the abstract of the talk can be found.

[2] All coffee breaks will be in the Red Centre, 3rd Floor, Staff Common Room.

# Tuesday 1 October

| | |
|---|---|
| **09:15–10:15** | **Olivier Ramaré**, *Products of three primes in arithmetic progressions and generalization*, page 8 |
| **10:15–10:35** | Coffee Break |
| **10:35–10:55** | **Kam Hung Yau**, *Product of prime and square-free integer in arithmetic progression*, page 16 |
| **11:00–11:20** | **Stephan Baier**, *Prime powers dividing products of consecutive integer values of* $x^{2^n}+1$, page 9 |
| **11:25–11:35** | Break |
| **11:35–11:55** | **Changhao Chen**, *On large values of Weyl sums*, page 10 |
| **12:00–12:20** | **Simon Macourt**, *Double Kloosterman sums in finite fields*, page 13 |
| **12:30–14:00** | Lunch |
| **14:00–14:20** | **Peter Humphries**, *Small scale equidistribution of lattice points on the sphere*, page 12 |
| **14:25–14:45** | **Ben Moore**, *Gauss sums and reciprocity*, page 14 |
| **14:50–15:05** | Group Photo |
| **15:05–15:25** | Coffee Break |
| **15:25–15:45** | **Jorge Mello**, *On integral points of semigroup orbits*, page 14 |
| **15:50–16:10** | **Ray Li**, *effective multiplicative dependence among iterated values of polynomials modulo finitely generated groups*, page 13 |
| **16:15–16:25** | Break |
| **16:25–16:45** | **Min Sha** *Functional graphs generated by plane curves and their twists*, page 15 |
| **16:50–17:10** | **Alexander Bors**, *Classification of finite 2-groups with exactly three automorphism orbits*, page 9 |

## Wednesday 2 October

| | |
|---|---|
| **09:15–10:15** | **Dzmitry Badziahin**, *Simultaneous Diophantine approximation on the Veronese curve*, page 7 |
| **10:15–10:35** | Coffee Break |
| **10:35–10:55** | **Ayreena Bakhtawar**, *Generalisations to Jarník-Besicovitch set via continued fractions*, page 9 |
| **11:00–11:20** | **Mumtaz Hussain**, *Measure theoretic frameworks for limsup sets*, page 12 |
| **11:25–11:35** | Break |
| **11:35–11:55** | **Wataru Takeda**, *On the Brocard-Ramanujan problem for homogeneous polynomials*, page 16 |
| **12:00–12:20** | **Richard Brent**, *The multiplication table problem revisited*, page 10 |
| **12:30–14:00** | Lunch |
| **14:00–14:20** | **Tianxin Cai**, *A New Generalization of Fermat's Last Theorem*, page 10 |
| **14:25–14:45** | **Catalin Hanga**, *Random matrix theory models for Gram's law*, page 11 |
| **14:50–15:10** | Coffee Break |
| **15:10–15:30** | **Asim Patra**, *The growth rate of random balancing sequence*, page 15 |
| **15:35–15:55** | **Forrest Francis**, *Small improvements in the Burgess inequality*, page 11 |
| **16:00–16:20** | **Peng Gao**, *Mean values of divisors twisted by quadratic characters*, page 11 |
| **18:30–** | **Conference Dinner** |

# Thursday 3 October

| | |
|---|---|
| **09:15–09:35** | **Randell Heyman**, *On a sum involving the Euler function*, page 12 |
| **09:40–10:00** | **Bryce Kerr**, *Large values of Dirichlet polynomials*, page 12 |
| **10:00–10:20** | Coffee Break |
| **10:20–10:40** | **Gopal Krishna Panda**, *Triangular-like sequences with all triangular numbers*, page 15 |
| **10:45–11:05** | **Sasha Fish**, *Patterns in sets of positive density in trees and buildings*, page 11 |
| **11:10–11:30** | **Enrique Treviño**, *On Egyptian fractions of length 3*, page 16 |
| **11:35–11:45** | Break |
| **11:45–12:05** | **Alexandru Ghitza**, *Fine structure of Hecke eigensystems mod p*, page 11 |
| **12:10–12:30** | **Thomas Morrill**, *Overpartition crank moments and quasimodularity*, page 14 |
| **12:40–14:00** | Lunch |

## ABSTRACTS OF INVITED TALKS

### Recent results around the Prime Geodesic Theorem

**Antal Balog**
**Alfréd Rényi Institute of Mathematics, Budapest**

**Abstract**. An element $P \in \Gamma = PSL(2, \mathbb{Z})$ acts on the upper half of the complex plane as a fractional linear transformation

$$Pz = \frac{az + b}{cz + d}.$$

By definition $P$ is hyperbolic if this transformation has two distinct real fixed points. $P$ is called primitive, if it is not a full power of other hyperbolic elements. The theorem in the title counts $\pi_\Gamma(x)$, the number of primitive hyperbolic elements $P$ satisfying $NP \le x$, with an appropriate notion of a norm $NP$. $\pi_\Gamma(x)$ has an asymptotic formula similar to the Prime Number Theorem. The similarity is transparent by understanding that in this rather artificial setting the role of the Riemann $\zeta$–function is played by the Selberg zeta–function

$$Z(s) = \prod_P \prod_{k=0}^{\infty} \left(1 - \frac{1}{(NP)^{s+k}}\right),$$

where $P$ runs over the set of primitive hyperbolic elements of $\Gamma$. Here the story becomes fascinating, because not only the Selberg zeta–function has meromorphic continuation, and functional equation, but it also satisfies the Riemann Hypothesis. All of these were known for Selberg in the 1950's. We discuss the development of this interesting subject.

**Simultaneous Diophantine approximation on the Veronese curve**

**Dzmitry Badziahin**
**University of Sydney**

**Abstract**. We call a point $\boldsymbol{\xi} \in \mathbb{R}^n$ simultaneously $\lambda$-well approximable, if for any $\epsilon > 0$ the inequality

$$\|q\boldsymbol{\xi}\| < Q^{-\lambda-\epsilon}, \quad 1 \le q \le Q$$

has integer solutions for infinitely many $Q \in \mathbb{Z}$. The point $\boldsymbol{\xi} \in \mathbb{R}^n$ is uniformly well approximable if the same inequality holds for all large values of $Q$.

The set of (uniformaly) well approximable points has a very complicated fractal-type structure. In this talk we will discuss what is known about it and especially about the intersection of this set with the Veronese curve $\mathcal{V}_n = \{x, x^2, \ldots, x^n : x \in \mathbb{R}\}$. This curve is particularly important because it it will provide us with some knowledge about the approximation by alrgebraic numbers.

Joint work with *Yann Bugeaud partly, partly myself and some results in the talk are made without me*

## Products of three primes in arithmetic progressions and generalization

**Olivier Ramaré**
**Aix Marseille Université**

**Abstract**. It is possible to mix additive combinatorics together with sieve methods to show that every invertible residue class modulo $q$ contains a small product of exactly three prime factors. We present the steps of such a proof and how to generalize it to ray-class group.

## ABSTRACTS OF CONTRIBUTED TALKS

1. **Stephan Baier** (*RKMV Educational and Research Institute, India*) *Prime powers dividing products of consecutive integer values of* $x^{2^n}+1$

   Abstract: Let $n$ be a positive integer and $f(x) := x^{2^n}+1$. We study orders of primes dividing products of the form $P_{m,n} := f(1)f(2)\cdots f(m)$. We prove that if $m > \max\{10^{12}, 4^{n+1}\}$, then there exists a prime divisor $p$ of $P_{m,n}$ such that $\mathrm{ord}_p(P_{m,n}) \leq n \cdot 2^{n-1}$. For $n = 2$, we establish that for every positive integer $m$, there exists a prime divisor $p$ of $P_{m,2}$ such that $\mathrm{ord}_p(P_{m,2}) \leq 4$. Consequently, $P_{m,2}$ is never a fifth or higher power. This extends work of Cilleruelo who studied the case $n = 1$.

   Joint work with *Pallab Kanti Dey*

2. **Ayreena Bakhtawar** (*La Trobe University*) *Generalisations to Jarník-Besicovitch set via continued fractions*

   Abstract: From Lagrange's (1770) and Legender's (1808) results we conclude that to find good rational approximations to an irrational number we only need to focus on its convergents. Let $[a_1(x), a_2(x), \ldots]$ be the continued fraction expansion of a real number $x \in [0, 1)$. Then Jarník-Besicovitch set in terms of continued fraction, consists of all those $x \in [0, 1)$ which satisfy

   $$a_{n+1}(x) \geq e^{\tau(\log|T'x|+\cdots+|T'(T^{n-1}x)|)}$$

   for infinitely many $n \in \mathbb{N}$, where $a_n(x)$ is the $nth$ partial quotient of $x$ and $T$ is the Gauss map. In this talk, we focus on determining the Hausdorff dimension of the set of real numbers $x \in [0, 1)$ such that for any $m \in \mathbb{N}$ the following holds for infinitely many $n \in \mathbb{N}$

   $$a_{n+1}(x)a_{n+2}(x)\cdots a_{n+m}(x) \geq e^{\tau(x)(f(x)+\cdots+f(T^{n-1}(x)))}$$

   where $f$ and $\tau$ are positive continuous functions. This set is stronger in the sense that for $m = 1$, $\tau(x)$ =constant and $f(x) = \log|T'|(x)$ it implies Jarník-Besicovitch set.

   Joint work with *Mumtaz Hussain*

3. **Matteo Bordignon** (*UNSW Canberra at ADFA*) *Explicit bounds on exceptional zeroes of Dirichlet L-functions II*

   Abstract: We will improve the upper bound for the exceptional zeroes of even Dirichlet L-functions. The result is obtained by improving on the explicit estimate for $L(\sigma;\chi)$ for $\sigma$ close to unity and $\chi$ even, using a result on the average of Dirichlet characters, and on the lower bound for $L(1;\chi)$, with computational aid.

4. **Alexander Bors** (*University of Western Australia*) *Classification of finite* 2*-groups with exactly three automorphism orbits*

   Abstract: Let $G$ be a finite group, and consider the natural action of the automorphism group $\mathrm{Aut}(G)$ on $G$. Denote by $\omega(G)$ the number of distinct orbits of this action. Since every automorphism of $G$ fixes the neutral element $1_G$, one has $\omega(G) = 1$ if and only if $G$

is trivial, i.e., $G = \{1_G\}$. Not quite on the same level of triviality, but still relatively easy is the proof that $\omega(G) = 2$ if and only if $G \cong (\mathbb{Z}/p\mathbb{Z})^d$ for some prime $p$ and some positive integer $d$. However, the classification of finite groups $G$ with $\omega(G) = 3$ is still open. In 1986, Laffey and MacHale classified those finite groups $G$ with $\omega(G) = 3$ and which are *not* of prime power order, so the problem is reduced to $p$-groups. In this talk, we discuss a recently achieved classification of the finite 2-groups $G$ with $\omega(G) = 3$, which uses a good amount of (elementary) number theory.

Joint work with *Stephen P. Glasby*

5. **Richard P. Brent** (*ANU and University of Newcastle*) *The Multiplication Table Problem Revisited*

Abstract: The *multiplication table problem* of Erdős concerns the asymptotic behaviour of the function $M(n)$ that counts the number of distinct products in an $n \times n$ multiplication table, i.e. $M(n) = \#\{i \times j : 1 \leq i, j \leq n\}$. We describe some algorithms for evaluating $M(n)$ in (quadratic) time $O(n^2)$, and a new algorithm that takes subquadratic time $O(n^2/L^{1/\sqrt{2}+o(1)})$, where $L = \exp(\sqrt{\ln n \ln\ln n})$. These algorithms have been used to compute $M(n)$ for various $n \leq 2^{30}$, extending the entries in OEIS A027417. We also describe two Monte Carlo algorithms, the *Bernoulli* and *product* algorithms, for estimating $M(n)$. These algorithms are practical for much larger $n$, and can be used to confirm the theoretical asymptotic behaviour [Ford, 2008] of $M(n)$ (convergence is very slow). In comparing the efficiencies of the Bernoulli and product algorithms, we were led to consider a function $T(n)$, defined to be the number of products that occur exactly twice in the $n \times n$ table. Clearly $T(n) = O(M(n))$. We present some evidence that $M(n) = O(T(n))$ (equivalently, that $T(n) \asymp M(n)$). For further details and numerical results, see arXiv:1908.04251.

Joint work with *Carl Pomerance, David Purdum and Jonathan Webster*

6. **Tianxin Cai** (*Zhejiang University, China*) *A New Generalization of Fermat's Last Theorem*

Abstract: We raised a new kind of Diophantine equation with both additive and multiplicative equations. In this talk we will discuss a new generalization of Fermat's Last Theorem, i.e., $x + y = z$, $xyz = w^n$, which is equal to FLT when $\gcd(x, y, z) = 1$. We will discuss it in different cases, including quadratic fields, and the recent attempt toward the possible proof of non-solution conjecture.

7. **Changhao Chen** (*UNSW Sydney*) *On large values of Weyl sums*

Abstract: We first give a survey on some known results about the Weyl sums. Then we show that for almost all $(x_1, x_2 \ldots, x_d) \in [0,1]^d$ the Weyl sums satisfy the upper bound

$$\left| \sum_{n=1}^{N} e^{2\pi i(x_1 n + x_2 n^2 + \ldots + x_d n^d)} \right| \leq N^{1/2+o(1)}, \quad N \to \infty.$$

We will also show that the exceptional set $(x_1, x_2, \ldots, x_d)$, which has large values of Weyl sums for infinitely many $N$, are large in the terms of Baire categories and Hausdorff dimension. In the end we give some new bounds of Weyl sums which improve the results of Wooley (2015).

Joint work with *Igor Shparlinski*

8. **Forrest J. Francis** (*UNSW Canberra at ADFA*) *Small improvements in the Burgess inequality*

   Abstract: Let $\chi$ be a Dirichlet character modulo $p$, a prime. It is often important to have an estimate for the size of sums of the form $\sum_{n=M+1}^{M+N} \chi(n)$, where $N < p$. There are two standard estimates in this direction. One, the *trivial bound*, depends entirely on $N$. The second, the *Pólya-Vinogradov inequality*, depends entirely on $p$. Between these two is the family of hybrid estimates known collectively as the *Burgess inequality*. In this talk, we present several straightforward techniques which yield minor improvements for the leading constants in the Burgess inequalities.

9. **Sasha Fish** (*University of Sydney*) *Patterns in sets of positive density in trees and buildings*

   Abstract: What are the patterns that necessarily occur in sets of positive density in homogeneous trees and certain affine buildings? What is the relationship between the frequencies of occurrence of elementary finite configurations in a tree and its growth rate? We will address both of these questions.

   Joint work with *Michael Björklund, James Parkinson and Leo Jiang*

10. **Peng Gao** (*Beihang University, China*) *Mean values of divisors twisted by quadratic characters*

    Abstract: In 2000, J. B. Conrey, D. W. Farmer, and K. Soundararajan determined completely the asymptotic formula for all $X, Y > 0$ of the following character sum:

    $$\sum_{m \leq X, n \leq Y} \left(\frac{m}{n}\right).$$

    It is relatively easy to obtain such an asymptotic formula when $X, Y$ are far apart in size. However, things become much subtler when $X$ and $Y$ are of comparable size. In this talk, we consider an analogue case by establishing the mean values of divisors twisted by quadratic characters. Other similar results will also be discussed.

11. **Alexandru Ghitza** (*University of Melbourne*) *Fine structure of Hecke eigensystems mod $p$*

    Abstract: Bergdall and Pollack recently determined dimension formulas for the $\overline{\rho}$-component of the spaces of modular forms of levels divisible by $p$, where $\overline{\rho}$ is a given mod $p$ Galois representation. On the other hand, Martin used trace formula methods to compute the dimensions of the Atkin–Lehner plus and minus eigenspaces.
    We report on work in progress on a common refinement of these results. In particular, we describe our computer experiments and how their results can be elucidated by considering the structural properties of modular forms mod $p$.

    Joint work with *Samuele Anni and Anna Medvedovsky*

12. **Catalin Hanga** (*University of York*) *Random Matrix Theory models for Gram's Law*

Abstract: Gram's Law refers to a pattern that often occurs in the distribution of the non-trivial zeros of the Riemann zeta function. Although this pattern doesn't hold for every zero, numerical results suggest that, as the height up the critical line increases, the proportion of zeros that satisfy Gram's Law converges to a non-trivial limit. And because it is well-know that the eigenvalues of certain kind of random matrices provide a good statistical model for the zeros of the zeta function, one could try to determine the value of this limit and its rate of convergence by analyzing an analogous model for Gram's Law in the framework of Random Matrix Theory. In this talk, I will review an existing model based on random unitary matrices, for which the limit can be computed analytically, but has the wrong convergence rate. I will then present an alternative model that uses random special unitary matrices, which gives the correct rate of convergence, and discuss about whether or not this model has the same limit as the previous one. The talk is designed to be understandable without any background knowledge in Random Matrix Theory.

Joint work with *Chris Hughes*

13. **Randell Heyman** (*UNSW Sydney*) *On a sum involving the Euler function*

Abstract: We obtain reasonably tight upper and lower bounds on the sum $\sum_{n \leqslant x} \varphi(\lfloor x/n \rfloor)$, involving the Euler functions $\varphi$ and the integer parts $\lfloor x/n \rfloor$ of the reciprocals of integers. For slower growing arithmetic functions $f$ we obtain asymptotic formulas for similar sums of $f(\lfloor x/n \rfloor)$. These are analogues of a series of previous results for sequences involving the integer part functions such Beatty and Piatetski–Shapiro sequences. Time permitting we will also discuss searching for primes in the set $\{\lfloor x/n \rfloor,\ n \leq x\}$.

Joint work with *O. Bordellés, L. Dai, H. Pan and I. E. Shparlinski*

14. **Peter Humphries** (*University College London*) *Small Scale Equidistribution of Lattice Points on the Sphere*

Abstract: Consider the projection onto the unit sphere in $\mathbb{R}^3$ of the set lattice points $(x_1, x_2, x_3) \in \mathbb{Z}^3$ lying on the sphere of radius $\sqrt{n}$. Duke and Schulze-Pillot showed in 1990 that these points equidistribute on the sphere as $n \to \infty$. We study a small scale refinement of this theorem, where one asks whether these points equidistribute in subsets of the sphere whose surface area shrinks as $n$ grows. A particular case of this is a conjecture of Linnik, which states that for all $\varepsilon > 0$, the equation $x_1^2 + x_2^2 + x_3^2 = n$ has a solution with $|x_3| < n^\varepsilon$ for all sufficiently large $n$. We make nontrivial progress towards this, as well as proving an averaged form of this conjecture.

Joint work with *Maksym Radziwiłł*

15. **Mumtaz Hussain** (*La Trobe University*) *Measure theoretic frameworks for limsup sets*

Abstract: We survey some fundamental measure theoretic methods that provide conditions under which limsup subsets (of a compact metric space equipped with a non-atomic probability measure) has positive or full measure.

16. **Bryce Kerr** (*UNSW Canberra at ADFA*) *Large values of Dirichlet polynomials*

Abstract: In this talk we show how ideas from the sum-product problem may be used to obtain new estimates for the number of large values of Dirichlet polynomials. Current

approaches to the sum-product problem establish relations between various energies using geometric incidences. To estimate the number of large values of exponential sums, one may proceed in analogy to sum-product estimates given a suitable replacement for geometric incidences. In the case of Dirichlet polynomials, this role is played by a generalization of Heath-Brown's convolution estimate. Our arguments build on work of Bourgain, Heath-Brown, Huxley and Jutila.

17. **Ethan Lee** (*UNSW Canberra at ADFA*) *Chebotarëv's Density Theorem*

    Abstract: We introduce the Chebotarëv density theorem. In order to find (effective) explicit bounds for this — without assuming the generalised Riemann hypothesis for Dedekind zeta-functions — we must seek effective upper and lower bounds individually. To this end, this talk will introduce Lagarias–Montgomery–Odlyzko's method for finding an effective lower bound and applications of better lower bounds.

18. **Ray Li** (*UNSW Sydney*) *Effective Multiplicative Dependence among Iterated Values of Polynomials modulo Finitely Generated Groups*

    Abstract: Let $\mathbb{K}/\mathbb{Q}$ be a number field and let $f(X) \in \mathbb{K}(X)$ be a rational function of degree at least 2. Northcott's Theorem shows the set of $\mathbb{K}$-rational preperiodic points has finite size. Berczes, Ostafe, Shparlinski and Silverman extend this to the case where points in an orbit need not be equal, but are multiplicatively dependent modulo a finitely generated subgroup $\Gamma$ of $\mathbb{K}$. Under mild assumptions they show only finitely many $\alpha \in \mathbb{K}$ can have such points in their orbit. However, their proof crucially relies on a Diophantine approximation result obtained using Roth's Theorem, hence is not effective. We consider the problem of obtaining effective results by replacing this crucial theorem by one obtained using Baker's Method. We obtain an effective and rather explicit variant of the result over $\mathbb{Z}$ and effective variants of the result in the case of a number field.

    Joint work with *Igor Shparlinski*

19. **Sean B. Lynch** (*UNSW Sydney*) *The error term in the prime ideal theorem for noncommutative arithmetic*

    Abstract: We begin by introducing arithmetic orders, our setting for noncommutative arithmetic. Then we investigate the distribution of maximal left ideals in arithmetic orders, generalising the distribution of prime numbers in the integers. In 1982, Bushnell and Reiner proved a main term of $\frac{x}{\log x}$ in the asymptotic formula for the number of maximal left ideals of index at most $x$ in an arithmetic order. We discuss a method for finding an error term in the asymptotic formula based on reducing to the centre of a maximal order containing the given order.

    Joint work with *Daniel Chan*

20. **Simon Macourt** (*UNSW Sydney*) *Double Kloosterman Sums In Finite Fields*

    Abstract: We define the Kloosterman sum over arbitrary finite fields

$$K_\psi(u,v) = \sum_{x \in \mathbb{F}_q^*} \psi(ux + vx^{-1}).$$

and we consider double sums of Kloosterman sums

$$S_\psi(u,v) = \sum_{u\in\mathcal{U}}\sum_{v\in\mathcal{V}} K_\psi(u,v)$$

over some subsets $\mathcal{U},\mathcal{V}\subseteq\mathbb{F}_q$. We give bounds on such sums where one, or both, of $\mathcal{U}$ and $\mathcal{V}$ are affine subgroups. Our results are dependent on recent bounds of additive energy over finite fields. The motivation behind this problem comes from recent advances in estimating various bilinear Kloosterman sums which have found applications to various arithmetic problems.

Joint work with *Igor Shparlinski*

21. **Jorge Mello** (*UNSW Sydney*) *On Integral Points of Semigroup Orbits*

    Abstract: A famous integrality result due to Siegel says that genus one curves have a finite number of integral Solutions. In this talk we will recall some analogous results in the context of arithmetic dynamics, with special interest for integrality in orbits of semigroups of rational functions over number fields.

22. **Ben Moore** (*University of Adelaide*) *Gauss sums and reciprocity*

    Abstract: In 1850, Mathias Schaar discovered an analytic proof of quadratic reciprocity. His proof involved, as an intermediate step, a remarkable identity between objects now known as Gauss sums. This identity was rediscovered by Landsberg in 1893 and is now known as the Landsberg-Schaar relation. We present some elementary proofs of the Landsberg-Schaar relation, together with some evaluations of twisted Gauss sums, and discuss generalisations of this picture in the context of automorphic forms and higher reciprocity laws.

23. **Thomas Morrill** (*UNSW Canberra at ADFA*) *Overpartition Crank Moments and Quasimodularity*

    Abstract: The Andrews-Garvan partition crank function gives the combinatorial interpretation of Ramanujan's partition congruence $p(11n+6)\equiv 0 \mod 11$, as predicted by Freeman Dyson. As with Dyson's rank function, the moment generating functions of the crank may be expressed as linear combinations of quasimodular forms. Recall, a quasimodular form is the holomorphic part of a nearly holomorphic modular form. We extend the notion of crank to overpartitions – partitions in which the first occurence of each integer part may be overlined – and study the quasimodularity of the resulting moment generating functions. This builds on prior work by Bringman-Lovejoy-Osburn.

    Joint work with *with Aleksander Simonic*

24. **Takashi Nakamura** (*Tokyo University of Science*) *Symmetric Tornheim double zeta functions*

    Abstract: Let $s,t,u\in\mathbb{C}$ and $T(s,t,u)$ be the Tornheim double zeta function. We investigate some properties of symmetric Tornheim double zeta functions. As a corollary, we give explicit evaluation formulas for $T(s,t,u)$ in terms of series of the gamma function and the Riemann zeta function.

25. **Gopal Krishna Panda** (*National Institute of Technology Rourkela, India*) *Triangular-like Sequences with all Triangular Numbers*

Abstract: A balancing-like sequence is a recurrence sequence satisfying the recurrence relation $x_{n+1} = Ax_n - x_{n-1}$ with initial terms $x_0 = 0$ and $x_1 = 1$, and $A > 2$ is a positive integer. For any given $A$, the $n$-th triangular-like number is defined by $\tau_n(A) = \frac{x_n \cdot x_{n+1}}{A}$. $\tau_1(A) = 1$ and hence is a triangular number for all $A$. $\tau_2(A) = A^2 - 1$ is a triangular number only if $A$ is an almost balancing number of second kind and these numbers are of the form $B_n - 2B_{n-1}$ or $2B_n - B_{n-1}$, where $B_n$ is the $n$-th balancing number. If $A > 2$, both $\tau_2(A)$ and $\tau_3(A)$ are triangular numbers only if $A = 4$. Furthermore, all the triangular-like numbers corresponding to the balancing-like sequence with $A = 4$ are triangular numbers. Moreover, the problem of finding the values of $A$ with the property that $\tau_3(A)$ is a triangular number leads to solving the Diophantine equation $8A^4 - 24A^2 + 17 = B^2$. This equation has only three known solutions $(A, B) = (1, 1), (2, 7), (4, 41)$, and has possibly no more solutions in positive integers.

Joint work with *S.S. Pradhan*

26. **Asim Patra** (*National Institute of Technology Rourkela, India*) *The Growth Rate of Random Balancing Sequence*

Abstract: In the present paper, the random balancing sequence is defined by the binary recurrence $x_{n+1} = 6\alpha x_n - x_{n-1}$, where $\alpha$ is a random variable which assumes the values $+1$ and $-1$ with probability $1/2$ each. In this work an elementary proof regarding the bounds of the expected value for the absolute value of the $n$-th term in the random balancing sequence has been provided. Moreover, the bounds for the variance of the absolute value of the $n$-th term has also been obtained. Furthermore, a generalization has been done for the random balancing-like sequence.

Joint work with *GOPAL KRISHNA PANDA*

27. **Min Sha** (*UNSW Sydney*) *Functional graphs generated by plane curves and their twists*

Abstract: Given a plane curve defined by $Y^2 = f(X)$ over a finite field $\mathbb{F}_q$ of odd characteristic and a non-square element $\lambda$ in $\mathbb{F}_q$, we define a functional graph by choosing the elements in $\mathbb{F}_q$ as vertices and drawing an edge from $x$ to $y$ if and only if $(x, y)$ is either a point on the curve $Y^2 = f(X)$ or a point on the curve $\lambda Y^2 = f(X)$. We show that if $f$ is a permutation polynomial over $\mathbb{F}_q$, then every connected component of the graph has a Hamiltonian cycle. Moreover, these Hamiltonian cycles can be used to construct balancing binary sequences. By making computations for permutation polynomials $f$ of low degree, it turns out that almost all these graphs are strongly connected, and there are many Hamiltonian cycles in such a graph if it is connected.

Joint work with *Joint with Bernard Mans, Jeffrey Smith, Daniel Sutantyo*

28. **Aleksander Simonič** (*UNSW Canberra at ADFA*) *Explicit Selberg's zero density estimate*

Abstract: In 1946, A. Selberg proved that $N(\sigma, T) \ll T^{1-\frac{1}{4}(\sigma-\frac{1}{2})} \log T$, uniformly for $\sigma \in [1/2, 1]$, where $N(\sigma, T)$ denotes the number of zeros $\rho = \beta + \mathrm{i}\gamma$ of the Riemann zeta-function

$\zeta(s)$ with $\beta > \sigma$ and $\gamma \in (0, T]$. In this talk we will see how to make each step in Selberg's proof explicit. As a by-product we also obtain

$$\int_0^T \left|\zeta\left(\frac{1}{2} + \mathrm{i}t\right)\right|^2 \mathrm{d}t = T \log T + (2\gamma - 1 - \log 2\pi) T + O\left(T^{\frac{3}{4}} \sqrt{\log \frac{T}{2\pi}}\right)$$

with known constants in the $O$-term.

29. **Wataru Takeda** (*Nagoya University*) *On the Brocard-Ramanujan problem for homogeneous polynomials*

    Abstract: The Brocard-Ramanujan problem, which is an unsolved problem in number theory, is to find integer solutions $(x, l)$ of $x^2 - 1 = l!$. Brocard and Ramanujan conjectured that the only solutions are $(x, l) = (5, 4), (11, 5)$ and $(71, 7)$ independently. As one of its generalization, Erdős and Obláth considered the equation $x^m \pm y^m = l!$ and proved that for $m \geq 3$ the equation $x^m + y^m = l!$ has no solution with $\gcd(x, y) = 1$ except for $(x, y, l) = (1, 1, 2)$ and $x^m - y^m = l!$ has no solution except for $m = 4$. For the case $m = 4$, Pollack and Shapiro showed that $x^4 - y^4 = l!$ also has no solution.
    We study an equation $F(x, y) = \Pi_K(l)$, where $F$ is a homogeneous polynomial with integer coefficients and $\Pi_K$ is a generalized factorial function over number fields $K$. We show a necessary condition for the existence of infinitely many solutions. As a corollary, we obtain the finiteness of $l$ such that $l!$ is represented as $F(x, y)$, where $F$ is irreducible or satisfies some condition. We also prove that there exist number fields $K$ and polynomial $F(x, y)$ such that for infinitely many $l$ the factorial $\Pi_K(l)$ is represented as $F$.

30. **Enrique Treviño** (*Lake Forest College*) *On Egyptian fractions of length 3*

    Abstract: Let $a, n$ be positive integers. We say $a/n$ can be represented as an Egyptian fraction of length $k$ if there exist positive integers $m_1, m_2, \ldots, m_k$ such that $\frac{a}{n} = \frac{1}{m_1} + \cdots + \frac{1}{m_k}$. In talk we will describe a parametrization for Egyptian fractions of length 3 and we apply this parametrization to give bounds to $A_3(n) = \#\{a : \frac{a}{n} = \frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}\}$, to $f_a(n) = \#\{(m_1, m_2, m_3) : \frac{a}{n} = \frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}\}$, and finally to $F(n) = \#\{(a, m_1, m_2, m_3) : \frac{a}{n} = \frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}\}$.

    Joint work with *Cyril Banderier, Carlos Alexis Gómez Ruiz, Florian Luca, Francesco Pappalardi*

31. **Tim Trudgian** (*UNSW Canberra at ADFA*) *Mean-square for free for PNT!*

    Abstract: Assuming the Riemann hypothesis, we have good bounds on the error term in the prime number theorem. No surprises: the *average* error term is much better than the *point-wise* error term. I shall outline some open problems in this area, and some current work with Dave Platt.

    Joint work with *Dave Platt, University of Bristol*

32. **Kam Hung Yau** (*UNSW Sydney*) *Product of prime and square-free integer in arithmetic progression*

Abstract: A conjecture of Erdös, Odlyzko and Sárközy asks if for every reduced residue class $a$ modulo $m$ can be represented as a product of two primes, each no more than $m$. In this talk, we will review the history and partial progress on this problem.

## LIST OF REGISTRANTS

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|---|---|
| Lucas Allen | ANU |
| Dzmitry Badziahin | University of Sydney |
| Stephan Baier | RKMV Educational and Research Institute, India |
| Ayreena Bakhtawar | La Trobe University |
| Antal Balog | Alfred Renyi Institute of Mathematics |
| Alexander Bors | University of Western Australia |
| Matteo Bordignon | UNSW Canberra at ADFA |
| Phillip Bos | La Trobe University |
| Richard Brent | University of Newcastle and ANU |
| Tianxin Cai | Zhejiang University, China |
| Changhao Chen | UNSW Sydney |
| Michaela Cully-Hugill | UNSW Canberra at ADFA |
| Matthew Di Meglio | UNSW Sydney |
| Alexander Fish | University of Sydney |
| Forrest Francis | UNSW Canberra at ADFA |
| Peng Gao | Beihang University, China |
| Alex Ghitza | University of Melbourne |
| Catalin Hanga | University of York |
| David Harvey | UNSW Sydney |
| Shehzad Hathi | UNSW Canberra at ADFA |
| Randell Heyman | UNSW Sydney |
| Peter Humphries | University College London |
| Mumtaz Hussain | La Trobe University |
| Danesh Jogia | Defence |
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| Kijung Kim | Pusan National University, Korea |
| Ethan Lee | UNSW Canberra at ADFA |
| Ray Li | UNSW Sydney |
| Sean Lynch | UNSW Sydney |
| Simon Macourt | UNSW Sydney |
| Jorge Mello | UNSW Sydney |
| Benjamin Moore | University of Adelaide |
| Thomas Morrill | UNSW Canberra |
| Gerry Myerson | Macquarie University |
| Takashi Nakamura | Tokyo University of Science |
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| Min Sha | UNSW Sydney |

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| Nicole Sutherland | University of Sydney |
| Aleksander Simonič | UNSW Canberra at ADFA |
| Omar Soukieh | Defence |
| Wataru Takeda | Nagoya University, Japan |
| Enrique Treviño | Lake Forest College, USA |
| Tim Trudgian | UNSW Canberra at ADFA |
| Chenyan Wu | University of Melbourne |
| Kam Hung Yau | UNSW Sydney |
| Yinan Zhang | ANU |
| Liangyi Zhao | UNSW Sydney |