Erratum for "Generalising Group Algebras", J. Lond. Math. Soc. (2) 72 (2005) 742–762

Hendrik Grundling

Department of Mathematics, University of New South Wales,

Sydney, NSW 2052, Australia.

hendrik@maths.unsw.edu.au FAX: +61-2-93857123

Math. subject class.: 46L05, 22D25, 43A10

Abstract

The main result in [2], the existence and uniqueness theorem for host algebras, is wrong.

The existence and uniqueness theorem for host algebras, Theorem 3.4 in [2], is wrong for the following reasons.

In Definition 3.1 we wanted to generalise the concept of an open projection away from universal enveloping Von Neumann algebras, so that it makes sense for any Von Neumann algebra $\mathcal{N}$. We defined a projection $P \in \mathcal{N}$ as open, if $\mathcal{N}P$ is the intersection of left kernels of normal states of $\mathcal{N}$. Unfortunately, *every* projection $P \in \mathcal{N}$ will satisfy this criterion by Theorem 3.6.11 in Pedersen [3], and its proof. In particular, if $\mathcal{N} = \mathcal{A}''$ for a C*–algebra $\mathcal{A}$, then this definition of an open projection does *not* define the usual open projections (cf. Proposition 3.11.9 [3]). Hence the proof of Theorem 3.4 in [2], which rests on this, fails.

The proof of Theorem 3.4 [2] cannot be repaired, because there is no intrinsic way to define the open projections in $\mathcal{N} = \mathcal{A}''$ without the knowledge of the subalgebra $\mathcal{A} \subset \mathcal{N}$. In fact, the knowledge of the open projections in $\mathcal{N}$ will uniquely determine the subalgebra $\mathcal{A} \subset \mathcal{N}$ by the subsequent argument in the proof of Theorem 3.4 [2], using the characterisation of $M(\mathcal{A})_{sa}$ as q–continuous elements of $\mathcal{N}$ in [1]. Since one can find a different C*–subalgebra $\mathcal{B} \subset \mathcal{A}''$ such that $\mathcal{B}'' = \mathcal{A}''$ and $\mathcal{B} \neq \mathcal{A}$ (e.g. by unitary conjugation of $\mathcal{A}$ in $\mathcal{A}''$), we see that the open projections w.r.t. the two subalgebras $\mathcal{A}$, $\mathcal{B}$ must differ, but as they produce the same Von Neumann Algebra $\mathcal{N} = \mathcal{B}'' = \mathcal{A}''$, their open projections cannot be defined purely from $\mathcal{N}$ itself, without the knowledge of how the subalgebras are included in $\mathcal{N}$.

This mistake affects most of Sect. 3, as well as the isomorphism claim in line 2, p745. The rest of the paper is not affected by it, nor as far as the author knows, has it compromised any other published results.

Acknowledgements.

I wish to thank Prof. Karl-Hermann Neeb for pointing out this error.

References

[1] Akemann, C.A., Pedersen, G.K., Tomiyama, J.: Multipliers of C*-algebras. J. Funct. Anal. **13**, 277–301 (1973)

[2] Grundling, H.: Generalising group algebras. J. London Math. Soc. (2) **72** (2005) 742–762.

[3] Pedersen, G.K.: C*–Algebras and their Automorphism Groups. Academic Press, London 1989