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Using Ulam's method to calculate entropy and other dynamical invariants. (English summary)
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The paper proposes a new rigorous result in the estimation of the metric entropy of a smooth $C^{1+\gamma}$ transitive hyperbolic dynamical system $T\colon M\to M$, on a compact Riemannian manifold M.

It is known that the statistical properties of such a system are described by a "natural" measure, the Sinaĭ-Bowen-Ruelle (SBR) probability measure μ. The author obtains estimates of (i) the SBR measure, (ii) the μ-Lyapunov exponents of T, (iii) the rate of decay of correlations with respect to C^γ test functions, and (iv) the pressure (for repellers).

These estimates are proved when dim $M=2$ except for expanding systems where they are established in all dimensions. The method relies on the approximation of smooth hyperbolic dynamical systems by Markov chains. Let $(\mathcal{P}_n)_{n\in\mathbf{N}}$ be a sequence of Markov partitions such that $\lim_n \operatorname{diam}\mathcal{P}_n=0$. Define the matrix

$$(\mathcal{Q}_n)_{i,j}=\frac{m(A_{n,i}\cap T^{-1}A_n^j)}{m(A_n^j)},$$

where m is the volume and $A_{n,k}$ describes the generic atom of $\mathcal{P}_n$. Then Q_n has a largest eigenvalue ρ_n and a strictly positive associated eigenvector v_n. Then define the stochastic matrix $(P_n)_{ij}=(Q_n)_{ij}v_{n,j}/P_nv_{n,i}$: it has a unique fixed eigenvector p_n. Then $\mu_n(\cdot)=\sum_{i=1}^{k_n}p_{n,i}m(A_{n,i}\cap\cdot)/m(A_{n,i})$ converges towards μ, $\lambda_n=-\sum_{i,j=1}^{k_n}P_{n,i}(P_n)_{ij}\mathrm{Log}(Q_n)_{ij}$ estimates the sum of the positive exponents and the μ-entropy of the system is approximated by $h_n=\mathrm{Log}\,\rho_n+\lambda_n$.

These results expand the previous one obtained by the same author; examples illustrate the method. The interest of the method—in contrast to standard techniques using a single long orbit of the system—consists in using information from all regions of the phase space. Unfortunately, in dimensions larger than 2 the numerical construction of Markov partitions may be time-consuming or impossible.

Reviewed by *Bernard Schmitt*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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