New records for enumerating primes and personal latitude

David Harvey, UNSW Sydney

Rump session, ANTS XVII (2026), Groningen

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Eratosthenes $\approx$ 200 BCE (one RAM op $= \log N$ bit ops)

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Bit size of output (either as a list or bit array)

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- $N(\log\log N)^{1+o(1)}$ — **New heuristic algorithm**
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New personal latitude record

60° 48.825’ N

a fjord near Bergen, Norway
last week

Thanks for your attention!