Exam check list

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Overview

In this course you should have learnt

- the basic concepts in linear algebra.
- how to apply linear algebra in a variety of contexts.
- write simple proofs.

Permutations, groups, fields, determinant

In this topic you should know how to

- verify something is a group or group action.
- calculate with permutations.
- prove some results concerning the determinant using the permutation definition.
- use various properties of the determinant.
- use Cramer's rule for theoretical purposes.

Linearity

In this topic you should understand

- Compute sums and intersections of subspaces.
- Linear maps & ways of combining them i.e. linear combinations and composition, & how they mirror matrix operations
- Isomorphisms of vector spaces, and in what sense isomorphic vector spaces are "the same".

Direct sums

In this topic you should

- understand the external direct sum construction and how given an internal direct sum, it is isomorphic to the external direct sum.
- understand and use the general matrix representation thm i.e. the correspondence between matrices of linear maps and linear maps between direct sums.

Co-ordinates

In this topic you should

- understand what it means to put a co-ordinate system on a vector space.
- know how to represent a linear map with a matrix given co-ordinate systems on the domain and codomain.
- know how to compute bases for kernels and images of linear maps by using the representing matrix.
- know how to compute bases for vector space complements.
- know how to extract information using the dimension and rank-nullity theorem.
- be able to use change of co-ordinates to compute matrices representing rotations and reflections.

Invariant subspaces

In this topic you should

- understand how decomposition into invariant direct sums gives block diagonal forms.
- know the various criteria for a matrix to be diagonalisable.
- represent some linear endomorphisms with diagonal matrices by using co-ordinates.
- be able to use similarity invariants to show matrices are not similar.
- be able to use the Cayley-Hamilton theorem.

Discrete & continuous time systems

In this topic you should know how to

- compute powers and exponentials of matrices using the Jordan canonical form.
- solve some discrete times systems.
- solve some continuous times systems using exponentials of matrices and variation of parameters.

Jordan canoniocal form

In this topic you should

- understand that Jordan canonical forms give a complete similarity invariant.
- be able to use primary decomposition to extract information about e-spaces etc.
- be able to find the Jordan canonical of a square matrix and the corresponding change of co-ordinates matrix.
- understand the interplay between Jordan canonical forms, Jordan form tableaux, algebraic and geometric multiplicity of e-values & the subspaces $E_\lambda(n)$.

Inner product spaces

In this topic you should

- appreciate that inner products allow you to import many geometric techniques to a wide class of vector spaces.
- be able to compute orthogonal complements.
- be able to compute orthogonal projections via "Fourier decomposition" or solving the normal equation.
- be able to orthogonalise a basis using the Gram-Schmidt algorithm & hence QR-factorise a matrix.
- be familiar with the notions of dual spaces and how they can be used to generalise the notion of transpose and adjoints of matrices to more general linear maps.
- know how to setup a least squares problem and solve it via the normal equation.

Spectral theory

In this topic you should know how to

- unitarily diagonalise hermitian & more generally, normal matrices.
- use the spectral theorem to study conics and quadric surfaces.
- determine if a matrix defines a reflection & determine the hyperplane of reflection.
- determine if a matrix defines a rotation & determine the axis of rotation and angle.
- compute SVD