FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS AND STATISTICS

MATH5725

GALOIS THEORY

Session 2, 2011
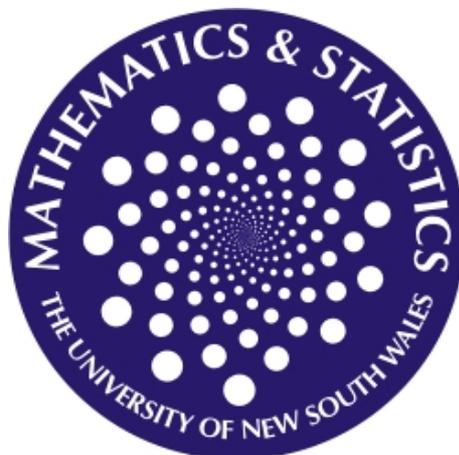
Course Outline

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Consultation Hours: TBA (see webpage)

Most of the information you need to know about the course can be gotten from the webpage above.

Lectures/Tutorials: In general, there will be 2 lectures a week, 1 tutorial a week. No classes will be held in the week of September 12-16. They will be replaced by classes in other weeks.

Lectures Notes: The course will be a simplified version of a course I gave four years ago. Boris Lerner has typed up notes to that course which should be available on the above webpage. Be warned that there will be substantial changes to them. Nevertheless, if you understand all the material in those notes, you should do fine for this course.

What you need to know to do this course

Ideal preparation for this course is the MATH3711: Algebra course. You need to be familiar with basic concepts in modern algebra and in particular, know a little about groups and field extensions. I hope the following notions are straightforward for you: normal subgroups, isomorphism theorems, group actions, minimal polynomials, algebraic extensions.

About this course

This course is a continuation of MATH3711 and, together with MATH5735: Modules and Representation Theory, rounds out the basic curriculum in undergraduate modern algebra. The material in this course is indispensable for anyone interested in number theory or algebra and is considered standard material for any pure mathematician.

The question which motivated Galois theory was: can you solve for the zeros of a polynomial in terms of the coefficients just using radicals, and the elementary operations of addition, subtraction, multiplication and division? The answer is yes for

quartic and lower order polynomials, but no for higher order. The proof requires a detailed study of the symmetry of field extensions obtained by adjoining roots of polynomials. This symmetry is encoded in a group called the Galois group. The main aim of this course is to study field extensions via the Galois group. In the process, you will learn about fundamental concepts in algebra such as, solvable groups and the Galois correspondence. You will also see Galois's proof of the impossibility of solving the quintic via radicals.

Assessment

The grade for this course will be determined from 2 short assignments (worth 15% each), and a final exam (worth 70%). The assignments are meant to be relatively straightforward, once you have understood the material. The hard part is of course understanding the material. It is expected that most of you will be getting close to full marks in the assignments. That way, a "pass" in the final exam should get you close to a credit. If you are having trouble with the assignments, you should talk to other students or to me about it. The most important thing is that you learn the material. The final exam will include questions of a more challenging nature and should distinguish the best students in the class.

Studying for this course

This course will be taught in a similar fashion to MATH3711. It is fairly demanding conceptually, but hopefully, you will be getting used to this. The concepts will take a while to digest so don't expect to understand everything in lectures. Try to get as much as possible out of them, and go over the material regularly after class. I suspect that filling in these gaps in understanding will take up a significant amount of your study for this course. If you are understanding very little of the lectures, then that's probably an indication that you haven't properly understood material in earlier lectures. I also strongly suggest you supplement your learning by browsing the references below.

As you progress in your academic career, you will be given fewer and fewer exercises to assist your learning. Many of you will find this difficult, especially if your study in the past consisted of doing homework problems, and consulting the teacher when you are unable to do them. To help wean you off homework problems, you should check out my MATH3711 handout on my webpage entitled "Studying for this course".

Student learning outcomes

Mathematically, I hope you will consolidate your understanding of the fundamentals of modern algebra and gain an appreciation of how deep abstract mathematical concepts can be used to solve concrete problems.

From a skills perspective, I hope you will develop your problem solving and analytical skills. The course should also help you improve your conceptual thinking. You

should understand by now, that modern mathematics is communicated in a very different fashion to other disciplines and to everyday speech. Though it can be terse, it has great precision. I hope in this course, you will gain a greater appreciation of the modes of mathematical communication.

References

The lectures will cover all the material that you need to know, but nevertheless, you will probably find it handy to supplement your studies by looking at texts such as those below. There are lots of texts designed for a first course in algebra. They vary a lot so you should scout around for what's suitable for you.

- Artin, Michael Algebra. Prentice Hall, Inc., Englewood Cliffs, NJ, 1991. xviii+618 pp. ISBN: 0-13-004763-5

- Bewersdorff, Jörg Galois theory for beginners. A historical perspective. Translated from the second German (2004) edition by David Kramer. Student Mathematical Library, 35. American Mathematical Society, Providence, RI, 2006. xx+180 pp. ISBN: 978-0-8218-3817-4; 0-8218-3817-2

- Cox, David A. Galois theory. Pure and Applied Mathematics (New York). Wiley-Interscience John Wiley & Sons, Hoboken, NJ, 2004. xx+559 pp. ISBN: 0-471-43419-1

- Escofier, Jean-Pierre Galois theory. Translated from the 1997 French original by Leila Schneps. Graduate Texts in Mathematics, 204. Springer-Verlag, New York, 2001. xiv+280 pp. ISBN: 0-387-98765-7

- Fraleigh, John B. A first course in abstract algebra. Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont. 1967

- Herstein, I. N. Abstract algebra. Third edition. With a preface by Barbara Cortzen and David J. Winter. Prentice Hall, Inc., Upper Saddle River, NJ, 1996. xviii+249 pp. ISBN: 0-13-374562-7

- Jacobson, Nathan Basic algebra. I. Second edition. W. H. Freeman and Company, New York, 1985. xviii+499 pp. ISBN: 0-7167-1480-9

- Kaplansky, Irving Fields and rings. Reprint of the second (1972) edition. Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 1995. x+206 pp. ISBN: 0-226-42451-0

- Stewart, Ian Galois Theory. Third edition. Chapman & Hall/CRC Mathematics. Chapman & Hall/CRC, Boca Raton, FL, 2004. xxxvi+288 pp. ISBN: 1-58488-393-6

- Stillwell, John Elements of algebra. Geometry, numbers, equations. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1994. xii+181 pp. ISBN: 0-387-94290-4

- Swallow, John Exploratory Galois theory. Cambridge University Press, Cambridge, 2004. xii+208 pp. ISBN: 0-521-83650-6; 0-521-54499-8

- Lang, Serge Algebra. Revised third edition. Graduate Texts in Mathematics, 211. Springer-Verlag, New York, 2002. xvi+914 pp. ISBN: 0-387-95385-X

- Weintraub, Steven H. Galois theory. Second edition. Universitext. Springer, NeW York, 2009. xiv+211 pp. ISBN: 978-0-387-87574-3

Continual Course Improvement

The School of Mathematics evaluates each course each time it is run. Feedback on the course is gathered using, among other means, UNSW's Course and Teaching Evaluation and Improvement Process. Student feedback is taken seriously, and continual improvements are made to the course based in part on such feedback.

School of Mathematics and Statistics Student Policies

School of Mathematics and Statistics policy regarding tests, assignments additional assessment etc can be found at

http://www.maths.unsw.edu.au/currentstudents/assessment-policies

The UNSW Plagiarism Policy is also there.

Daniel Chan