School of Mathematics University of New South Wales

MATH5725: Galois Theory (2016,S2)
Problem Set: Infinite Galois theory [1]

This problem sheet covers lectures on infinite Galois theory.

1. Let G be a profinite group. Show that G is a T_1-space i.e. all points are closed, but usually not Hausdorff.

2. Prove that $\hat{\mathbb{Z}} \simeq \prod \hat{\mathbb{Z}}_p$ where p runs through all the primes.

3. Let p, q be primes and G_q be the kernel of the canonical projection map $\hat{\mathbb{Z}} \longrightarrow \mathbb{Z}/q\mathbb{Z}$. What is the intermediate field of $\bar{\mathbb{F}}_p/\mathbb{F}_p$ corresponding to G_q? Let H be the closed subgroup of $\hat{\mathbb{Z}}$ defined by $H = \cap_q G_q$ where q ranges over all primes. What is the intermediate field corresponding to H?

4. Show that the only non-trivial intermediate fields of $\mathbb{F}_{p^{q^\infty}}/\mathbb{F}_p$ are the $\mathbb{F}_{p^{q^n}}$

5. Determine all the intermediate fields of $\bar{\mathbb{F}}_p/\mathbb{F}_p$.

[1] by Daniel Chan