Lecture 6: Linear Dependence cont'd

Testing Lin Independence in $\mathcal{R}[\mathbb{R}]$

No systematic method.

E.g. 1 $\{e^x, e^{2x}, e^{3x}, \ldots\}$ is lin indep

Why? Suppose $\lambda_1 e^x + \ldots + \lambda_n e^{nx} = 0$

with

How does Span(S) vary with S?

Propn V = vector space/ field $\mathbb{F}$

Let $S_1 \subseteq S_2$ be subsets of V. Then

Proof: Every element in Span(S_1)

is

so is

so is

i.e.

Thm 1 Let $S \subseteq V$ = vector space/ field $\mathbb{F}$

For $\mathbf{v} \in V$,

$$\mathrm{Span}(S \cup \{\mathbf{v}\}) = \mathrm{Span}(S)$$

iff $\mathbf{v} \in \mathrm{Span}(S)$.

Proof: ($\Longrightarrow$) If $\mathrm{Span}(S \cup \{\mathbf{v}\}) = \mathrm{Span}(S)$ then

($\Longleftarrow$) Suppose $\mathbf{v} \in \mathrm{Span}(S)$. Recall that $\mathrm{Span}(S \cup \{\mathbf{v}\})$ is the

But $\mathrm{Span}(S)$

cont'd

Hence, Span$(S) \supseteq$

Propn $\implies$ Span$(S) \subseteq$

The two inclusions show

Note: We saw if $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ lin depend then some $\mathbf{v}_i \in$ Span$(S - \{\mathbf{v}_i\})$. So then

i.e. can omit $\mathbf{v}_i$ without

E.g. 2 $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^3$ non-parallel.

Suppose $\mathbf{v}_3 \in W =$ Span$(\mathbf{v}_1, \mathbf{v}_2)$

Then Span($\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$)

Alternative algebraic way to see this:

Any lin combn

$x_1 \mathbf{v}_1 + x_2 \mathbf{v}_2 + x_3 \mathbf{v}_3 =$

$\therefore$ Span($\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$)

Propn $\implies$

so

argument can be jazzed up to prove thm 1 (see notes §6.4.4)

e.g. What's Span($1, x^2, 3 + x^2$)?

$3 + x^2 \in$

Informal First Look at Coordinates

$\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^3$ lin indep

$W =$ the plane

Set up “grid” or “coordinate” system

$\mathbf{v}_1, \mathbf{v}_2$ determines coords on W

$\mathbf{v}_1$ has coords

The following thm generalises the above to show "S lin indep determines coordinate system for vectors in Span(S)".

Thm (Uniqueness of Linear Combn)

V = vector space/ field $\mathbb{F}$

Let $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ non-empty, lin

Any vector $\mathbf{v} \in \text{Span}(S)$ can be written

$\mathbf{v} =$

i.e. If also $\mathbf{v} =$

then

Note: $\lambda_1, \ldots, \lambda_n$ give coordinates of $\mathbf{v}$.

Proof: Suppose

$\mathbf{v} =$

Subtracting using distributive, associative and commutative laws gives

Defn of lin indep $\implies$

What Fails if S Lin Dependent?