Lecture 3: Linear Combinations and Span

Aim of Lectures 3/4: Recall that a non-zero direction vector $\mathbf{v} \in \mathbb{R}^n$ determines a line through the origin.

2 non-parallel vectors $\mathbf{v}, \mathbf{w}$ determines a plane through the origin.

We generalise these ideas to arbitrary

Linear Combinations

Defn (copy defn for $\mathbb{R}^n$)

$V =$ vector space/ field $\mathbb{F}$

Consider subset S of V. A linear combn

cont'd

If $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_m\}$ then a lin combn is

E.g. 1 $\mathbf{v}_1 = (2, 1), \mathbf{v}_2 = (0, 1)$
an e.g. of

E.g. 2 $V = M_{22}(\mathbb{R})$

$$\mathbf{v}_1 = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix}, \mathbf{w} =$$

$\mathbf{w}$ is not a lin

Why?

Propn 1 If $S \subseteq W \leq V =$ vector space/ field $\mathbb{F}$, then every linear combn of elements in S is in W.

Proof: For $\mathbf{v}_i \in S$

$\lambda_1 \mathbf{v}_1 + \ldots + \lambda_r \mathbf{v}_r$

Span

Defn Span (copy defn for $\mathbb{R}^n$)

$V =$ vector space/ field $\mathbb{F}$

Consider subset S

cont'd

If $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_m\}$ then Span(S) is Span$(\mathbf{v}_1, \ldots, \mathbf{v}_m)$

Define Span $(\varnothing)$

Note $S \subset$

E.g. 1 Recall for $\mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ non-parallel

E.g. 2 Consider vector space $\mathbb{P}$.

Note: Both spans are subspaces.

Thm (Span is a subspace) Let $S \subseteq V =$

vector space/ field $\mathbb{F}$.

In fact, Span(S)

Proof: Check Span(S) is a subspace via subspace thm-defn.

a. Span(S) $\neq \varnothing$ for if $S = \varnothing$ then $\mathbf{0}$

while if

b. Closure under addn. Consider 2 typical elts of Span(S),

So Span(S) is

c. Closed under scalar multn. For $\lambda \in$

cont'd

So Span(S) is

Subspace thm-defn

We know now Span(S) is a subspace containing S so, for final assertion of thm, suffice show any subspace W containing S also contains Span(S). Propn 1

e.g. Let $\mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ be non-zero.

If $\mathbf{v}, \mathbf{w}$ not parallel

If $\mathbf{v}, \mathbf{w}$ parallel

Defn (Spanning Set) If $V = \mathrm{Span}(S)$ then we say that S spans V or that is a spanning set for V.

E.g. $\{1, x, x^2, \ldots\}$ is a spanning set for $\mathbb{P}$.