Lecture 23: Intro to Eigenbases

Inevitable Woffle

Q Why did we introduce abstract notion of vector spaces?

A 1. To handle infinite dim

2. Defn is

Let V = vector space/ field $\mathbb{F}$

Next few lectures, study lin maps of form

i.e. domain =

Aim $T : V \longrightarrow V$ often picks out its own preferred coord system/basis for V. Wish to describe this

E.g. 1 $V :=$ 3-dim space of

$T : V \longrightarrow V$ is rotatn about axis $L =$ Span $\mathbf{u}$ about angle θ.

N.B. Can check geom that T is

Let $P =$

Preferred coord system has L

i.e. if $\mathbf{w} \in P$ is

preferred basis $\mathcal{B} = \{$

W.r.t $\mathcal{B}$

E.g. 2 $V :=$ space of 2-dim

Let $L =$

Let $T : V \longrightarrow V$ be reflection about L

Let $S : V \longrightarrow V$ be orthog

Let $L' =$

Preferred Basis is

Matrix representing T is

Matrix representing S is

Key Notion The subspaces L, L', P above are examples of invariant subspaces.

Defn Let $T : V \longrightarrow V$ be lin. A subspace

W of V

In this case, restricting domain to W gives linear

E.g. L, L', P in

Remark The matrices above obtained using "preferred" bases have "block diagonal" form with blocks representing $T|_W$ where W

Eigenvectors

Optimal scenario when invariant subspaces are 1-dim so blocks have smallest size.

Q When's this occur?

Ans: $W = \operatorname{Span} \mathbf{v}$ is T-invariant iff

(*)

Defn (Eigenvector) If $\mathbf{v} \neq 0$ is as in (*) above we say that

E.g. 2 again

$\mathbf{u}, \mathbf{v}$ are

E-values are

$\lambda \quad T \qquad S$

$\mathbf{u}$

$\mathbf{v}$

E.g. 3

$$A = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}, T = T_A :$$

E.g. 4 $V = \mathcal{C}^\infty$

$T : V \longrightarrow V$ is differentiation.

Thm Let $T : V \longrightarrow V$ be

Suppose $B = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is

The matrix representing T wrt B is the diagonal matrix

Proof: Generalised matrix representation thm (lecture 11) shows cont'd

Change of Basis

Let A be an $n \times n$-matrix so T_A :

Let $B = \{\mathbf{f}_1,$

Q What's matrix rep T_A wrt

Let $M = (\mathbf{f}_1 \ldots \mathbf{f}_n)$

Thm 2 The matrix C rep

Proof: Need prove first

Lemma $S_B(\mathbf{v}) := [\mathbf{v}]_B =$

Proof: $\mathbf{f}_i = M\,\mathbf{e}_i = M[\mathbf{f}_i]_B$

$\therefore S_B(\mathbf{f}_i) =$

The linear maps S_B

Resume proof thm: $C = ([A\,\mathbf{f}_1]_B \ldots [A\,\mathbf{f}_n]_B)$

E.g. 5 Suppose $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ has evectors

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

with e-values 1,2. Find matrix A rep T.

Ans: