Lecture 22: Central Limit Thm.

Approximations.

Aim Look at some practical applications of the central limit thm to approx probabilities.

E.g. 1 X = weight of student

is a random var with mean $\mu = 70$ & std devn $\sigma = 10$.

15 students climb up Rap

which supports 1080 kg.

Find

Ans: Let $Y =$

Central Limit thm $\Longrightarrow$

i.e. $Y =$

where Z is approx

$P(Y < 1080) =$

$\approx \Phi(.7746) \approx .78$

E.g. 2 Effectiveness of weight loss diet.

Suppose $X =$

has std devn 3kg. How many people, say n, must you sample to ensure sample mean $\overline{\mathrm{X_n}}$ is 90% likely to be within .5 kg of

Ans: $\overline{\mathrm{X_n}} \approx$

i.e. $Z =$

is approx

Want $.9 < P(\mu - .5 < \overline{\mathrm{X_n}}$

i.e. need Φ

Tables $\Longrightarrow$

Approx to Binomial Distrbn

Recall, if Y has

$p_k =$

it is the same as the sum of n indep measurements of the dichotomous var

X with $P(X = 1) =$

Can apply central limit thm if n is large.

E.g. 3 60% of all e-mail is

In a sample size of 24, what's prob

Use normal approx.

Ans: Let $Y =$

Actual prob

$P(Y \geq 12) =$

is hard to calculate.

Recall $E(Y) =$

$\text{Var}(Y) =$

$\sigma(Y) =$

Central limit thm $\implies Y \approx$

or $Z =$

$P(Y \geq 12) = P(Z \geq$

Shifted Normal Approx

Last example used a cont prob distrbn to approx a discrete one. Doesn't distinguish

$P(Y > a)$ with $P(Y \geq a)$ (which are different in discrete case). We can "account" for this using half-shift method to improve approx.

E.g. 3 again

Area of rectangles better

If Z actually has std normal distribn then

$P(Y \geq 12) \approx P(Z \geq$

Poisson Approx to Binomial

Thm Let $\lambda = np$ be held

$$\lim B(n, p, k) =$$

Note: 1. Thm can be interpreted as meaning for large n & small p, the binomial

2. Approx useful $\because$ binomial coeff

Proof Thm: $B(n, p, k) =$

$$\frac{n(n-1)\cdots(n-k+1)}{n^k}\frac{n^k}{k!}p^k(1-p)^{n-k}$$

$$(1-\frac{\lambda}{n})^{n-k}$$

$\therefore B(n,p,k) \longrightarrow$

E.g. 4 Biased coin shows heads $p = .02$ of the time. Approx the prob of 4 heads in 100 tosses using the Poisson distribn.

Ans: Use Poisson with λ

$B(100, .02, 4) \approx$

Rules of Thumb

Normal Approx: usually OK if $n > 20$

$$np \;\&\; n(1-p) > 5$$

Poisson Approx: usually OK if $n > 50$

$$np \text{ or } n(1-p) < 5$$