# Lecture 21: Sums. Means. Central Limit Thm

**Aim** Compute $E(X), \mathrm{Var}(X)$ for sums & means of random var. Observe how all prob distribn lead to the normal distribn.

## Sums of 2 Random Var

Discussion in discrete case only

See later courses for cont case (need multivariate stats).

Consider 2 random var

$X$ with values

&

$Y$ with values

&

$X + Y$ is a discrete random var with values

Let $p_{kj} := P(X =$

Recall if $X, Y$ are indep

Also note

$p_k =$

**Propn 1** a) $E(X + Y) =$

b) If $X, Y$ are indep then

$\text{Var}(X + Y) =$

(c.f. Pythagoras) $\sigma(X + Y) =$

Proof: Discrete case only.

a) $E(X + Y) =$

cont'd

b) $E((X+Y)^2) =$

$\text{Var}(X+Y) =$

cont'd

**E.g. 1** The fat in mini chocolate bars is a random var normally distributed with $\mu$ =1g , $\sigma = .2$. Find mean & standard dev of total fat in 2 bars.

Ans: Let $X_1, X_2$

so $X_1 \sim$

mean total fat =

stand dev =

**Sums & Means**

Let $X$ be

Let $X_1, X_2, \ldots, X_n$ be

**E.g. 2** $X$ as in e.g. 1.

$X_1, X_2, \ldots, X_n$

**E.g. 3** $X_1, X_2 =$ temp in

Write $Y_n$ for

$\overline{\mathrm{X_n}}$

Latter often called sample mean.

**Propn 2** a) $E(Y_n) =$

b) $\mathrm{Var}(Y_n)$

c) $E(\overline{\mathrm{X_n}})$

d) $\mathrm{Var}(\overline{\mathrm{X_n}})$

Proof: a),b) follow from

c)

d)

Remark: You expect c) and also that $\mathrm{Var}(\overline{\mathrm{X_n}})$ shrinks

**E.g. 1 again** $X =$ fat in choc bars

$\mu = 1, \sigma = .2.$

Find the expected value and standard deviation of the sample mean of 16 bars.

## Dichotomous Var

**Defn** A dichotomous random var is one with

Often call 1 sucess and 0 failure.

**E.g. 4**

Write prob of success $p := P$

so $q :=$

**Propn 3** In this dichotomous case, $E(X) =$

Proof: $E(X) =$

$\mathrm{Var}(X) =$

**Binomial Distribn as a Sum of Dichotomous Var**

**E.g. 4 again** $X =$ coin toss.

$Y_n =$ sum of $n$ indep measurements

Hence $P(Y_n = k) =$

This viewpoint allows us to reprove

**Propn 4** If $Y$ has binomial prob distribn $B(n, p, k)$ then

a) $E(Y) =$

b)

Proof: $Y = Y_n$ in e.g. above. Using above notn

a) $E(Y_n) = nE(X) =$

b) $\mathrm{Var}(Y_n) = n\,\mathrm{Var}(X) =$

Remark: The proof of this propn in lecture 17 used generating fns.

**Central Limit Thms**

Let $X$ = random var

$Y_n$ = sum of $n$ indep measurements

$\overline{\mathrm{X_n}} =$

The central limit thm states that as $n \longrightarrow \infty$,

**Notn** If $Y$ is a random var write $Y \approx N(\mu, \sigma^2)$ if

$P(a < \sigma Z + \mu$

where $Z$

**Thm**(Central Limit)

With notn as above and $\mu = E(X), \sigma =$

a) For $n$ large,

$Y_n$

Recall $E(Y_n) = n\mu, \mathrm{Var}(Y_n) = n\,\sigma^2$.

b) For $n$ large,

Recall $E(\overline{\mathrm{X_n}}) = \mu, \mathrm{Var}(\overline{\mathrm{X_n}}) =$

c) If $X$ is normally distributed then

Remark: In fact, the error in approximating probabilities in a),b) tend to 0 as $n \longrightarrow \infty$

**E.g.** Binomial and dichotomous var