Lecture 20: Normal Probabilities.
Independent Random Variables.

Aim Compute probabilities of normally distributed variables. Introduce the notion of independent random variables.

Tables

Consider normal prob distribn:

$\phi(z) =$ $\Phi(z) =$

Note: Can't integrate $\phi(z)$ by usual calculus methods (the integral is not one of the standard elementary fns in first year calc courses).

Instead use table of values of $\Phi(z)$

E.g. 1 Let Z be a standard normal var.

Find $P(Z < .51)$.

For $z < 0$ use,

Propn 1 $\Phi(-z) =$

Why? The graph of $\phi(z)$

Computing Standard Normal Probabilities

E.g. 2 For a standard normal var Z find $P(-1.31 < Z < .51)$.

cont'd

Computing Normal Probabilities

E.g. 3 X = temperature in my dodgy freezer

Suppose X is normally distributed with $\mu = -2, \sigma = 3$.

What's the prob that temp is less than -3?

Ans: For Z stand normal var, $X \sim$

$P(X = 3Z - 2 <$

Good values of Φ to Know

Let Z have

Facts a) $P(-1 < Z < 1) \approx .68$

i.e.

b) $P(-2 < Z < 2) \approx .95$

c) $P(-3 < Z < 3) \approx .997$

Proof: Easy. For e.g. a)

Independence of Random Var

E.g. 4 X = roll

Y = no. shown on roulette

The value of X is not influenced by the value of Y. They are said to be

Note the probability that

$$P(X = 1, Y = 1) =$$

E.g. 5 X = temp in freezer as in e.g. 3.

X_1 = temp at

$X_2 =$

Both X_1, X_2 have the same

However, the value of X_1

In fact, X_1, X_2 should have more or less

Consequently,

$P(3.9 < X_1 < 4.1, 3.9 < X_2 < 4.1) \approx$

Considering the fact that

$E(X) =$ $\sigma(X)$

$P(3.9 < X_1 < 4.1)P(3.9 < X_2 < 4.1)$

Hence,

We say here that X_1, X_2 are dependent.

The above 2 examples suggests following mathematical defn of independence.

Defn (Independence)

2 random var X, Y are

if for any

$P(a < X < b, c < Y < d) =$

Propn 2 Consider discrete random var

X with values & prob distribn

Y with values & prob distribn

X, Y are independent iff

$P(X = x_k, Y = y_j) =$

Proof: $\implies$ easy.

Propn 3 Cont random var X, Y are independent iff

$P(X < a, Y < b) =$

Proof: omitted but easy.

E.g. 6 Suppose X, Y are discrete random var with values $\{x_1, x_2\}$ and

Suppose the probabilities $P(X = x_k, Y = y_k)$ are given in the table below as

$Prob$	x_1	x_2
y_1	.1	.3
y_2	.4	

Note $P(X = x_2, Y = y_2) =$

Are X, Y independent?

Ans: $P(X = x_1) =$

$P(Y = y_1) =$

Consider discrete random var X, Y

values x_k &

Get new random var XY with

values & $P(XY = w) = \sum$

Sim defn in cont case which we'll omit. The only fact we really need about XY is

Propn 4 Let X, Y be indep random var. Then $E(XY) =$

Proof: Only do discrete case. Let

X have values $\{x_k\}$ and prob distribn $\{p_k\}$.

Y have