Lecture 19: Scaling. Normal Distribution

Aim Introduce the most important distribution, the normal distribution.

Probability Density of $g(X)$

X cont random var with

Let $g(x)$ be increasing diff'ble

Lemma Let $Y = g(X)$ have prob density $f_1(y)$ where $y = g(x)$. Then

a) $F_1(y) = P(Y = g(X) <$

$P(X$

b) $f(x) = g'(x) f_1(g(x))$

Proof: a) holds as g

b) follows from

Important special case is

Scaling

We can scale X to $Y =$

where $a >$

Lemma $\implies f(x)$

or in other words

$f_1(y) =$

Recall from lecture 18

$E(Y) =$

Suppose graph of $f(x)$ is

Note Graph changes by

1) Mean

2) horizontal scale "stretches"

3) vertical scale

How does $\mathrm{Var}(X)$ change with scaling?

Propn 1 $\mathrm{Var}(aX + b) =$

$\sigma(aX + b) =$

Proof: $\mathrm{Var}(aX + b) = E((aX + b)^2) - E(aX + b)^2$

cont'd

$\sigma(aX + b) =$

Cor If X has mean μ standard deviation σ then $Y = \frac{X-\mu}{\sigma}$

A Useful Integral

Lemma

$$I := \int_{-\infty}^{\infty} e^{-z^2/2} dz = \sqrt{2\pi}$$

Proof: Consider solid of revolution on revolving $w = e^{-x^2/2}$ about w-axis.

This is graph of

$r =$

We'll compute volume under graph by

Slices: Fix x

Cross-sectional area $A(x) =$

Hence, volume

Cyclindrical Shells:

$$\text{Volume} = \int_0^\infty 2\pi r w dr$$

$\therefore I^2 =$

Standard Normal Distribn

Defn A cont random var Z has standard normal distribn if its prob density fn is

$\phi(z) =$

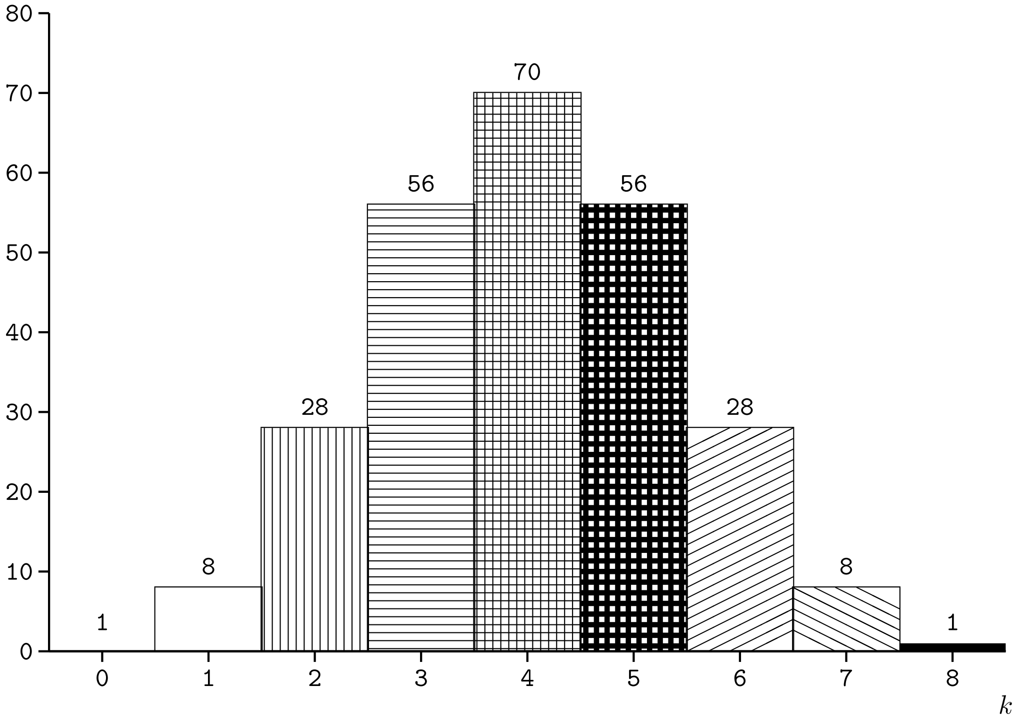
Note: 1. Lemma $\Longrightarrow$

2. The graph is symmetrical

Remark It is hard to say at the moment why the normal distribn is so important. We'll see later. Note for now that it has a similar bell shape to the binomial distribn except that it's continuous.

E.g. $B(n = 8, p = .5, k) =$

k	1	2	3	4	5	6	7	8
$256p_k$				70	56			



Propn Let Z be

a) $E(Z) =$ b) $\sigma(Z) =$

Proof: a) $\phi(z)$ even $\implies$

b) $\mathrm{Var}(Z) = E(Z^2) =$

Normal Distribn

We scale the standard normal variable Z to get

Defn A cont random var X has normal distribn if

Notn Write $X \sim N(\mu, \sigma^2)$.

Results on scaling show

Facts If $X \sim$

a) $E(X) =$ b) $\mathrm{Var}(X) =$