Lecture 17: Mean and Variance for Discrete Random Variables

Aim Lecture Define and compute the mean & variance for a discrete random var.

Recall for random var X on a population of size n, with values x_k occurring with freq f_k then,

$\overline{\mathrm{x}} =$

Since $\frac{f_k}{n}$

Defn (Expected Value) For X = discrete random var with

values

& probability distribn

the mean or

$\mu = E(X) =$

whenever the sum converges.

E.g. 1 Uniform distribn on $\{1, \ldots, n\}$.

$p_1 = \ldots$

$E(X) =$

Given a random var X and a fn $g : \mathbb{R} \longrightarrow \mathbb{R}$ we can consider a new random var $Y = g(X)$.

E.g. 2 $X =$ radius of

$Y =$ vol

$=$

Propn 1 Let X be a discrete random var with

Let $g : \mathbb{R} \longrightarrow \mathbb{R}$ be a fn and Y

Then $E(Y) =$

Proof: omitted.

Propn 2 i)For $a, b \in \mathbb{R}$,

$E(aX + b) =$

ii) For fns $g, h : \mathbb{R} \longrightarrow \mathbb{R}$,

$E(g(X) + h(X)) =$

Proof: i) Propn 1 $\implies$

$E(aX + b) =$

$= a \sum$

ii) Sim. omitted.

E.g. 3 Let $X =$ temp of Sydney in Celsius

$Y =$ temp of Sydney in

$E(Y) = E($

Recall for random var X on a population of size n, with values x_k occurring with freq f_k then the sample variance is

This suggests as before

Defn (Variance) For $X =$ discrete random var with values $\{x_k\}$ & probability distribn $\{p_k\}$, the

$\text{Var}(X) :=$

The standard deviation is

$\sigma(X) =$

Cor $\text{Var}(X) = E(X^2) - E(X)^2$.

Proof: $\text{Var}(X) = E((X - E(X))^2)$

N.B. In gen, you expect $E(X^2) \neq$

Generating Functions

The following method gives best way to find $E(X), \text{Var}(X)$ if values of X are $x_k = k$.

Defn For a probability distrbn $\{p_k\}$, its generating fn is

E.g. 1 Uniform distribn on $\{0, 1, 2\}$.

$p_0 =$

$p(t) =$

Propn 3 Let $p(t)$ be the generating fn for the prob distrbn for X with values $x_k = k$. Then

a. $E(X) =$

b. $E(X^2 - X) =$

c. $E(X^2)$

Proof: a) $p'(t) =$

$p'(1) =$

b) $p''(t) =$

$p''(1) =$

c) $E(X^2) = E(X^2 - X + X)$

We'll use this propn to compute $E(X)$ & $\text{Var}(X)$ for the Poisson & Binomial distrbn.

Poisson Distrbn with parameter λ

Recall $p_k =$

$p(t) =$

Propn 4 For the Poisson distrbn,

a. $E(X) = \lambda$ b. $\mathrm{Var}(X) = \lambda$.

Proof: a) $p'(t) =$

$E(X) =$

b) $p''(t) =$

$E(X^2) =$

$\mathrm{Var}(X) =$

Binomial Distrbn $n \in$

Recall $p_k = B(n, p, k) =$

Generating fn $p(t) =$

Propn 5 For the Binomial distrbn,

a. $E(X) = np$ (You know this!)

b. $\text{Var}(X) = np(1-p)$.

Proof: a) $p'(t) =$

$E(X) =$

b) Sim. Exercise.

E.g. 4 If X has Poisson distrbn with param $\lambda = 3$. What's the probability that X is below average?

Ans: Average $E(X) =$

$P(X <$

$(\approx .423)$

E.g. 5 Suppose a biased coin is tossed 64 times.

Let X = no. heads

$p_k =$

If sample variance is 12, what's the probability of all heads showing up assuming sample variance is actual variance?

$\text{Var}(X) = 12 =$

Solving for p gives $p =$

$\therefore$ prob $= p_{64} =$

E.g. 6 Let X be the number of times Gollum says "my precious" in any hour. X has Poisson distribn with $E(X) = 2$. Is the expected value greater than the median value?

$p_k =$

$p_0 + p_1 =$

$(\approx .406)$

$p_0 + p_1 + p_2 =$

$(\approx .677)$