Lecture 16: Intro to Probability & Random Variables

Aim Lecture Introduce some of the language of statistics.

E.g. 1 In statistics one might wish to examine:

a) trivia scores of

b) no. times someone has

c) high temperature

In each case have

1. A *population* which is

& 2. a *random variable* which is

E.g. 1 again

a) $\Omega =$

b) $\Omega =$

c) $\Omega =$

Defn A sample is a subset of

Probability

In above cases, Ω is finite, say with n elements.

Let im $X = \{x_1,$

i.e. range

Let $f_k =$

$=$ no. of elements in $\{$

If we pick a member $\omega \in \Omega$ at random, we expect that

$P(X = x_k)$

This prompts

Defn Let $X : \Omega \longrightarrow \{x_1, \ldots, x_r\}$ be a random var on a finite set Ω. The probability distribution

Properties of Prob Distr $\{p_k\}$

1. $p_k \geq 0$, 2.

Proof:

An Abstract Example

E.g. 2 $X : \Omega \longrightarrow \{2, 3, \ldots, 12\}$ is sum of values of

$\Omega = 36$

x_k	2	3	4	5	*etc.*
$P(X = x_k)$					

Q What if we change Ω here to all occurrences of 2 thrown dice in the past, present & future?

Ω is nebulous, possibly infinite and X cannot be computed. However, we would still like to say that

Soln In this case, we tend to ignore the precise role of Ω and make

There are two cases:

A. In E.g. 1 a),b), X takes on

B. In E.g. 2c), X takes on

Defn A random var is discrete if its values $\{\ldots, x_{-1}$

Defn A probability distribution for a discrete random var X as above is

satisfying

Note: We have seen that if $X : \Omega \longrightarrow \mathbb{R}$ is a random var defined on a finite set then

Uniform Distribution

Set of values of random var X is $\{x_k =$

$k|k \in \mathbb{Z}\}$. Let $S = \{y_1, \ldots, y_n\} \subset \mathbb{Z}$. Let

$p_k =$

Then $\{p_k\}$ is

Why?

What's the point? If $S = \{1, \ldots, 6\}$,

$X : \Omega \longrightarrow S$ is the roll

& if Ω is a typical population then $\{p_k\}$ models

Binomial Distribn

Fix first parameters $n \in$, $p \in$

Define

$p_k = B(n, p, k) =$

Note $p_k = 0$ if

Then $\{p_k\}$

Why?

What's this model? Suppose given a "biased" coin which shows

heads

& tails

Let $X =$ no.

What's $P(X = k)$?

Ans: The chance that the tosses $i_1, \ldots, i_k$ are heads and the others are tails is

There are possibilities

$P(X = k) =$

Poisson Distrbn

This models rare occurences. (See later for meaning of this).

Fix a parameter $\lambda > 0$. Define

$p_k =$

Then $\{p_k\}$

Why? a)

b) Need Taylor series from calculus

$e^x =$

So $\sum p_k =$

E.g. 3 Suppose

$X =$ no. aces

has Poisson distribn with parameter $\lambda = 2$.

N.B. This is a

Find the probability that more than 2 aces are served.

Ans: $P(X > 2) =$