Lecture 1: Vector Spaces

Aim of Today's Lecture:

Properties of $\mathbb{R}^n$: Write $\mathrm{V} = \mathbb{R}^n$, $\mathbb{F} = \mathbb{R}$.

For any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in$

1. Closure Under Addition:

2. Associative Law of Addition:

3. Commutative Law of Addition:

4. Existence of Zero:

5. Existence of Negatives:

6. Closure Under Scalar Multiplication:

7. Associative Law for Scalar Multn:

8. $1\mathbf{v} =$

9. Scalar Distributive:

10. Vector Distributive:

Defn of Vector Space

Let $\mathbb{F}$ be a field like $\mathbb{Q}, \mathbb{R}, \mathbb{C}$. A vector space over $\mathbb{F}$ is

a. A set V of

& b. An addition law denoted + which

This new vector is called

& c. A scalar multn law

satisfying

Properties 1-10 called

Eg Set $\mathbb{R}^n$ with

addition rule:

scalar multiplication rule:

Eg $\mathbb{C}^n$ is a vector space over

Define addn law as usual

Axioms

Eg Let $M_{mn}(\mathbb{F})$ be the set of $m \times n$-matrices over the field $\mathbb{F}$. Define

vector addition:

scalar multiplication:

You can check all axioms. Here we'll only check axiom 4:

Eg Let $\mathbb{P}$ be set of real polynomials.

$p(x) =$

$q(x) =$

Vector addn: $(p + q)(x) =$

Scalar Multn: For $\lambda \in$

Can check 10 axioms to

e.g. $\lambda(p(x) + q(x)) =$

Why vector spaces?

Vector spaces exhibit similar

Eg For vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ simplify $2\,\mathbf{v} + \mathbf{w} - 3\,\mathbf{v}$.

Eg For $m \times n$-matrices M, N simplify $2M + N - 3M$.

Only used

Conclusion: Much of the arithmetic for geometric vectors

Properties of Vector Spaces

Propn In a vector space V /

1. Uniqueness of Zero: The eqns in $\mathbf{w}$

(*)

has a unique soln denoted $\mathbf{0}$ & called

2. Cancellation:

3. Uniqueness of negatives: For any $\mathbf{v} \in V$, the eqn

We call

Proof: Do 1 only. Soln exists by axiom. Suppose $\mathbf{w} =$

Eg $M_{mn}(\mathbb{F})$. The unique negative of A

Propn Let V be a vector space over $\mathbb{F}$. For

$\lambda \in \mathbb{F}, \mathbf{v} \in V$

1. $\lambda \mathbf{0} =$ 2. $0\,\mathbf{v} =$ 3. $(-1)\,\mathbf{v} =$

4. $\lambda\,\mathbf{v} = \mathbf{0} \Longrightarrow$

Proof: Do 2 only.

$0\,\mathbf{v} + 0\,\mathbf{v} =$

Canceln

e.g. Twisted $\mathbb{C}^n$. $V =$

Addn:

New twisted scalar multn:

V is a vector

Why?