

## Lecture 9: Geometric & Coordinate Vectors

**Aim Lecture** In nature, sometimes only interested in the magnitude of a quantity e.g. mass, area. Sometimes, also interested

**Defn** A geometric vector in 3-dim (or 2-dim) space, usually denoted by bold symbol like  $\mathbf{v}$  (written

Usually represent vector  $\mathbf{v}$  by arrow of length

e.g.

**Degenerate case** Zero vector

**Rem** Often confuse arrows with

**Addition of geom vectors** for vectors  $\mathbf{v}$ ,  $\mathbf{w}$  define new vector  $\mathbf{v} + \mathbf{w}$  by picture below

i.e. slide  $\mathbf{w}$  so

then arrow of  $\mathbf{v} + \mathbf{w}$

N.B. Parallelogram above  $\implies$

i.e.  $\mathbf{v} + \mathbf{w} =$

**Scalar Multiplication** For vector  $\mathbf{v}$  &

scalar

If  $\lambda \geq 0$ ,  $\lambda \mathbf{v}$  is

If  $\lambda < 0$ ,  $\lambda \mathbf{v}$  is

**e.g.  $\mathbf{v}$**

**$\mathbf{v} + \mathbf{w}$**

**$\mathbf{v} + \mathbf{w}$  has mag**

**& is in dirn**

**What might this calculation represent physically?**

Banal **e.g.** (units = km) Frodo moves

**e.g.** Mumakil heads east

Legolas' arrow speed is

Total velocity

Calculation with vectors best performed after imposing a

**Coordinate System** in 3-dim (or 2-dim) space i.e.

a) point of

b) unit of

c) 3 (resp. 2) orthogonal

Axes usually satisfy

Given such a coord system, can represent a geom vector  $\mathbf{v}$  by its coord vector which is the triple  $(x, y, z)$  giving

Conversely, a triple  $(x, y, z)$  gives a vector

More gen

**$n$ -tuples** Let  $\mathbb{F}$  = field e.g.

An  $n$ -tuple ( in  $\mathbb{F}$ )

Let  $\mathbb{F}^n$  denote

**e.g.**  $\mathbb{R}^3$  is the set of coord vectors of

**Notn**  $\mathbf{v} = (v_1,$

where  $v_i$  are called the

**Addn in  $\mathbb{F}^n$**  For  $\mathbf{v} = (v_1, \dots, v_n)$

$\mathbf{w} =$

called the sum of

**Scalar Multn in  $\mathbb{F}^n$**  If  $\lambda$

define new  $n$ -tuple

**Fact** The sum & scalar multn of geom vectors are the

Why?

**Rem** Sometimes call  $\mathbf{v} \in \mathbb{F}^n$  a (coord) vector even if

**Defn** Given  $\mathbf{v} = (v_1, \dots, v_n) \in \mathbb{R}^n$  or

Its magnitude is  $|\mathbf{v}| :=$

**e.g.**  $3(2, -1, -1) +$

**e.g.**  $\mathbf{v} = (3, 4, 12), |\mathbf{v}| =$



N.B. In general, the magnitude of a triple in  $\mathbb{R}^3$  is the magnitude of

**Standard Basis Vectors** In  $\mathbb{F}^n$  define special vectors

called

If there's a coord system in 3-dim (or 2-dim) space, have corresponding geom vectors

In 3-dim space

2-dim space

**N.B.** geom vector  $a\mathbf{i}$