Lecture 7: Trigonometric identities from complex numbers

Aim Lecture Euler's formula suggests

Binomial Formula

Defn-Propn For $n \in, 0 \leq$

the binomial

It's the no. ways of picking

Binomial Thm

$(a+b)^n =$

Why?

Facts 1.$\overline{e^{i\theta}} =$

2. $\sin\theta =$

3. $\cos\theta =$

Proof From picture

2. (3. is similar)

e.g. 1 Write $\sin^4\theta$ in terms of cos

A $\sin^4\theta =$

$$= \frac{1}{\quad}(e^{i4\theta} - 4e^{i3\theta}e^{-i\theta} +$$

N.B. $\sin^4\theta$ is an even

Uses Can integrate

Rem Fourier theory (taught in 2nd yr) shows that any nice fn of period 2π can be expanded as above using constants, $\cos\theta$,

Conversely, $\sin n\theta, \cos n\theta$ can be converted back to a polynomial

e.g. 2 Write $\sin 3\theta, \cos 3\theta$

A De Moivre $\implies$

$\cos 3\theta + i \sin 3\theta$

Equate

$\cos 3\theta =$

$\sin 3\theta =$

The answer is not

$\cos 3\theta = \cos^3$

Bizarre Application

Solve $4z^3 - 3z = \frac{\sqrt{3}}{2}$

A Suppose there is a soln of form $z = \cos$

So $4z^3 - 3z = \cos$ $\qquad$, $\frac{\sqrt{3}}{2} =$

Hence $3\theta =$

See algebra notes Ch 1,ex.61 for more info.

Method generalises to higher order poly if you use "elliptic" fns.

Sums of trig fns

e.g. 3 Find

$\Sigma := \cos\theta + \cos 3\theta + \cos(2n+1)\theta.$

A $\Sigma =$

But the sum of a

$$e^{i2\theta} - 1 =$$

$$e^{i(2n+3)\theta} -$$