

Lecture 6: Solving polynomial equations

Aim Lecture The use of polar forms makes it

Square Roots

e.g.1 Solve $z^2 = -5 + 12i$

A Let $z = a +$

Equate real & imag parts

Solve simultaneously by guessing or elim a
or better still

$$|z^2|$$

Quadratic eqn Quadratic formula still holds.

e.g.2 Solve $z^2 + (-4 + i)z + (5 - 5i) = 0$.

quadratic formula:

discriminant =

From e.g.1 we see

Hence,

Roots of complex numbers

e.g.3 Find all cube roots of $8i$.

A Suppose $z^3 = 8i, z = r$

Polar form of $8i =$

$$z^3 = r$$

Equate moduli:

Equate arg:

$$\text{N.B. } -\pi < \theta \leq \pi \implies$$

Hence,

So $z =$

Note: 3 roots. In lecture 8 we will see

Let's plot the 3 cube roots

Cardano's Formula for the cubic eqn

$$z^3 + az + b = 0$$

is $z = u - \frac{a}{3u}$ where

$$u = \left[-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}} \right]^{1/3}$$

Proof Suppose $z = u - v$

Case $b = 0$