Lecture 5: Euler's formula & "applications"

Aim Lecture Multn & computing powers/roots best

Lemma $(\cos\theta + i\sin\theta)(\cos$

Proof LHS $=$

remark Here, mult numbers correponds to

Geom Interpretn

$z = \cos\phi + i\sin\phi, w = r($

zw "is" w

Euler's Formula For $\theta \in$

More gen,

$e^{a+bi} :=$

e.g. $e^{i\pi/2}$

Q Why is this defn sensible?

A 1. It extends defn for reals & in fact you'll learn in 2nd yr complex analysis courses that it's

2. We have the desirable

Formula $e^{z+w} =$

For $z =$

Proof LHS $=$

RHS $=$

3. In MATH1241 you'll see following formulae

$$e^x = \quad 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots$$

$\sin x =$

$\cos x =$

Suggests

$$e^{ix} = 1 + ix + i^2\frac{x^2}{2!} + i^3\frac{x^3}{3!} + i^4\frac{x^4}{4!} \dots$$

Formulae For $n \in \mathbb{Z}$

1. $e^{i(\theta+2n\pi)} =$ 2. $(e^{i\theta})^n =$

Proof 1. holds as $\cos, \sin$

2. Case a), $n \geq 1$

$(e^{i\theta})^n =$

Case b), $n = 0$

Case c), $n < 0$

$e^{in\theta}e^{-in\theta} =$

so $e^{in\theta} =$

An immediate corollary is

De Moivre's Thm

Proof

e.g. $e^{i\pi/2} = i, i^2 = -1 \implies$

Powers of complex numbers

Q Find $(\sqrt{3} - i)^{100}$

Dumb method

Good Method: Write $z = \sqrt{3} - i$ in

$|z| =$

$\operatorname{Arg} z =$

$z =$

$z^{100} =$

Formulae For $z, w \in$

1. $|\frac{z}{w}| =$
2. $\operatorname{Arg} zw =$

3. $|z^n| =$
4. $\operatorname{Arg} z^n$

Proof half of 1 & 2 only. Let $z = re^{i\theta}, w =$

$\frac{z}{w} =$

Hence, $|\frac{z}{w}| =$

Arg

e.g. Let $z = -1 + i, w = 1 + \sqrt{3}i$. Find argument and modulus of zw.

e.g. What's $|\frac{\bar{z}}{z}|$?