Lecture 3: Fields or Systems of Numbers

Q In what sense is $\mathbb{C}$ with addn

Aim Lecture Answer this question by

Fields: Let $\mathbb{F}$ be a set.

E.g.

Imprecise Defn $\mathbb{F}$ is a field (technical name for system of numbers) if

More precisely, suppose

i) an addn rule on $\mathbb{F}$ assigning

ii) a multn rule

Say $\mathbb{F}$ (with these 2 rules) is a field if it satisfies

For any $x, y, z \in \mathbb{F}$

1. Associative Law of Addition:

2. Commutative Law of Addition:

3. Existence of Zero:

4. Existence of Negatives:

5. Associative Law of Multn:

6. Commutative Law of Multn:

7. Existence of One:

8. Existence of Inverses: If $x \neq$

9. Distributive Law:

E.g. $\mathbb{F} = \mathbb{Q}$,

Defn Laws 1-9 are called

Subtrn/Division [H] Using 4. we can define subtraction as adding the negative. Sim division is

Subtle Point [H]: Need uniquesness of negatives and inverses to

Thm [H]: 1. $\mathbb{F} = \mathbb{C}$ is

2. Subtrn and division in $\mathbb{C}$ is given by adding the negative and multiplying

Proof 1. Need to check 9 axioms. We'll only check axiom 2.

2. We'll check subtrn only.

$0 = 0 + 0i$ is the zero since

The negative of $c + di$ is $-(c + di) =$

Let's add the negative of $c + di$ to

This is our defn

Manipulating complex numbers To multiply, divide etc. complex numbers, we usually use

e.g. 1 (3+i)(3-i)

2. $(1{+}\mathrm{i})^2 =$

The 9 laws above yield lots of other formulae we are familiar with in the algebra of real numbers. E.g.

Formula For $w, x, y, z \in \mathbb{C}$

Proof[H] $(\frac{w}{x})(\frac{y}{z}) =$

Need $x^{-1}z^{-1} =$

We check this

We usually use this formula to divide

e.g. 3

We have more

Formulae For $w, z \in$

1. $\bar{\bar{z}}$
2. $Re\ z =$
3. $\overline{z - w}$
4. $\overline{zw}$
5. z is real iff

Some **Proofs**

e.g. 4 Show that $\bar{z}w + z\bar{w}$ is real.

e.g. 5 Solve the following for z.

$z^2 + iz = yz.$