Lecture 26: Distances between lines, points & planes

Aim Lecture The notions of projection and cross product are useful for

Distance from point to plane Let P be point in $\mathbb{R}^3$. Let V be plane containing pt A with normal vector $\mathbf{n}$.

Drop a perpendicular from P to Q on V. Q is called the orthogonal

Pythagoras $\implies$ Q is

The distance of P to V is

To compute this note the rectangle above $\implies$ $\vec{QP} =$

e.g. 1 Find the dist from pt $P(2,2,2)$ to the plane V with Cart eqn $x_1 + 2x_2 - x_3 = 2$.

Ans We use notn as above. V contains pt $A(2,0,0)$ & has normal $\mathbf{n} =$

$\vec{AP} =$

If Q is orthog projn of P onto V then

$\vec{QP} = \text{proj}_{\mathbf{n}}$

$=$

$=$

$\therefore$ dist of P to V is

Distance between two lines in $\mathbb{R}^3$

Skew Case Let L, L' be skew lines in $\mathbb{R}^3$.

The shortest lines segment PQ from L to L' is

$\therefore$ if A is a pt on L, B pt on L' & $\mathbf{n}$

Dist L to $L' = |\vec{PQ}| = |$

e.g. 2 Find shortest dist between lines

$L : \mathbf{x} = (1, 0, 0) + \lambda(1, 2, 1), \lambda \in \mathbb{R}$

$L : \mathbf{x} = (0, 1, 0) + \lambda(1, 1, 2), \lambda \in \mathbb{R}$

Ans In notn above, can take A to be

& $\mathbf{n} = (1,$

$= (4 - 1,$

$\therefore \vec{PQ} = \text{proj}_{\mathbf{n}}$

$=$

$\therefore$ dist between lines $= |\vec{PQ}| =$

Case of Parallel Lines

Let L, L' be parallel lines & P a pt on L.

The dist between L & L' is also

$\because$