Lecture 25: Planes in $\mathbb{R}^3$.

Aim Lecture Planes in $\mathbb{R}^3$ are determined by a pt on them &

This yields

Cross product connects this with the

Point Normal Form Let $V \subset \mathbb{R}^3$ be a plane

Let $\mathbf{c}$ be (coord vector of) pt on V & $\mathbf{n}$

Then coords of any pt $\mathbf{x}$ on V satisfy

(*)

by picture above. In fact, pts of V are given precisely as

(*) is called the

N.B. There are lots of choices for

Converting param to pt-normal form

e.g.1 Let V be plane defined param by $\mathbf{x} = (2,1,2) + \lambda(1,-2,3) + \mu(4,-5,6)$

In pt-normal form can pick $\mathbf{c} =$

For normal $\mathbf{n}$, pick any scalar mult of

$(1,-2,3)$

$= (-12 + 15,$

$= 3(1,$

Pick $\mathbf{n} =$

Pt-normal form is:

Pt-Normal to Cartesian form

e.g. 2 Consider plane in pt-normal form $(1,2,1).(\mathbf{x} - (2,1,2)) = 0$. Find Cart form.

Ans $(1,2,1).\,\mathbf{x} -$

$(1,2,1).(x_1,$

This is the Cart form.

Param to Cart form As promised in lecture 16, have new way of making this conversion in 3-dim case. Just apply

e.g.3 The Cart form of

$\mathbf{x} = (2, 1, 2) + \lambda(1, -2, 3) + \mu(4, -5, 6)$ is

e.g. 4 Find Cart eqn of plane V containing $O(0, 0, 0), A(1, 1, 2), B(0, 3, 2)$.

Ans In pt-normal form, let $\mathbf{c} =$

& $\mathbf{n} = \vec{OA} \times$

$=$

$\therefore$ pt-normal form is

$\therefore$

Cart to pt-normal form

If $\mathbf{n} = (n_1, n_2, n_3)$ normal to plane V

& $\mathbf{c}$

$\mathbf{n} . (\mathbf{x} - \mathbf{c}) = 0 \implies \mathbf{n} . \mathbf{x} =$

i.e. $n_1 x_1 +$

Upshot The components of $\mathbf{n}$ are the

e.g.5 Find pt-normal form for

$3x_1 + 2x_2 + x_3 = 7.$

Ans