Lecture 2: Arithmetic of Complex Numbers

Aim of Lecture: Enlarge set of

Q What is

Possible **A** A real number

Motivation for complex numbers

Q Suppose graph of a polynomial fn is

e.g. $y = x^2 + bx + c$

What's d?

A

In e.g. quadratic formula $\Longrightarrow$

Problem: If

However, if we pretend the expressions exist & obey

Ex If $y = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots a_n$ has an axis of symmetry then it is

Conclusion: It is often useful in maths to pretend expressions such as

Later will see more serious applications of this idea. The above expressions don't represent amounts but are artificial

Today's **Q** What expressions should we treat as numbers and how

N.B. If usual laws of

Defn Let i

A complex number is an

Let $\mathbb{C}$ denote

e.g.

N.B. $\mathbb{C}$ contains the set of

Q When are 2

A Assuming usual

$a + bi = c + di \implies$

since

Suggests

Defn $a + bi = c + di$ means

Defn Let $z = a + bi, a, b \in \mathbb{R}$ be a complex number. The real

Complex numbers are equal iff

The complex conjugate

e.g.

Q What are sensible ways to add,

For complex numbers $z =$

Addn Define

e.g.

Subtrn Define

e.g.

Multn A priori not even clear

Let's see what happens if

Suggests defn

Neat Formula $z\bar{z}$

Proof

Division Assume $w \neq 0$ i.e.

Need trick to suggest formula for

Suggests defn

Note c^2