Lecture 17: Linearity of Soln set. Practical applications.

Aim Lecture In lecture 13, saw solns to lin eqns in 3 real var forms either i) $\varnothing$ ii) pt iii) line iv) plane v) all $\mathbb{R}^3$. This lecture examines

Also look at a practical applicn of lin eqns.

Parametric descrn of soln set Let $\mathbb{F}$ be a field

Let A be an $m \times n$-matrix & $A\,\mathbf{x} = \mathbf{b}$ system of lin eqns $\mathbf{b} \in$

Gaussian elimn shows if there is

$$\mathbf{x} = \mathbf{x}_p + \lambda_1 \mathbf{v}_1 +$$

where $\mathbf{x}, \mathbf{v}_1, \ldots, \mathbf{v}_s \in \mathbb{F}^n$ &

$s =$ no.

N.B. 1. If $\mathbb{F} = \mathbb{R}, s = 1, \mathbf{v}_1 \neq$

2. If instead $s = 2, \mathbf{v}_1, \mathbf{v}_2$ non-

3. For arbitrary s, soln set still "linear". This lecture, we'll justify this statement by showing by direct (& useful) argument that if $\mathbf{x}_1, \mathbf{x}_2$ are distinct solns, then so is line containing them and if $\mathbf{x}_3$ 3rd soln not collinear then

Some matrix laws $A = (a_{ij})$ be an $m \times n$-matrix (here a_{ij} is the

Let $\mathbf{x} =$

so $n =$ no. columns of A. Recall,

$$A\,\mathbf{x} = \begin{pmatrix} a_{11}x_1 + \ldots + a_{1n}x_n \\ \vdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n \end{pmatrix}$$

e.g.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} 7 \\ 8 \end{pmatrix}$$

Laws for $\mathbf{v}, \mathbf{w} \in \mathbb{F}^n$

Distributive law $A(\mathbf{v} + \mathbf{w}) =$

Formula For $\lambda \in \mathbb{F}$, $A($

Proof Follows from defns, see notes §3.7. Here just do distr law in 2×2 case. Gen case sim. Let $\mathbf{v} =$

$$\text{LHS} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}(v_1 + w_1) & a_{12} \\ a_{21}(v_1 + w_1) & a_{22} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}v_1 + a_{12}v_2 \\ a_{21}v_1 + a_{22}v_2 \end{pmatrix} +$$

$$= A\,\mathbf{v} +$$

Defn The homogeneous eqn corresp to

Solns called

(Easy) **Homog Case** Consider eqn $A\mathbf{x} = \mathbf{0}$.

1 We always have the soln

2 Given solns $\mathbf{x} = \mathbf{v}, \mathbf{w}$ then

3 Given soln $\mathbf{x} = \mathbf{v}$ & $\lambda \in$

Proof 1. By direct substn or something stupid like

$A\mathbf{0} =$

2.

3.

Rem a) 3 means the whole "line"

b) 2,3 & induction show given homog solns $\mathbf{v}_1, \ldots, \mathbf{v}_s$ &

In particular, the plane

Inhomog case

Propn Consider solns $\mathbf{x} = \mathbf{x}_1, \mathbf{x}_2$ to $A\,\mathbf{x} = \mathbf{b}$. Then $\mathbf{x}_2 =$

for some homog

Conversely, for any $\mathbf{w}$

So given any particular soln

$\{\mathbf{x}_p +$

Proof Let $\mathbf{v} = \mathbf{x}_2 -$

$A\,\mathbf{v} =$

so $\mathbf{v}$ is an

If $\mathbf{w}$ is an homog soln then

$A(\mathbf{x}_p + \mathbf{w}) =$

so $\mathbf{x}_p + \mathbf{w}$

Rem a). Suppose $\mathbf{x}_1, \mathbf{x}_2$ distinct inhomog solns. Then line containing

Fact 3) of homog case $\implies$ all these pts

2. Sim given 3 non-collinear solns $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$,

A practical applicn of lin algebra

E.g. You have captured

He tells you the ingredients are

Q 1. What are proportions of each?

2. Can you tell if he's lying?

To answer, need some facts about ingredients &

Analyse potion: Suppose 100mL contains

$b_1 = 5$g Nitrates, $b_2 = 10$g K-salts, $b_3 = 9$g

Mg Salts & $b_4 = 3$g Chlorophyll.

Ingredients also contain these compounds.

	N	K	Mg	Chl
$Mistletoe$	1	1	1	1
Oak	2	3	2	2
Eye	1	3	3	0

amount in g of compound in each 100g of ingredient. Let

$x_1 =$ amount of

$x_2 =$ amount of

$x_3 =$ amount of

Each compound gives an eqn. Hence system of lin eqns

$$(A|\,\mathbf{b}) =$$

cont'd

Soln is $\mathbf{x} =$

N.B. If you couldn't solve or if some value of $x_i <$