Lecture 16: Geometric Applications. Case of Unique Solns

Aim Lecture Solve some problems arising from chapter 2 material using

e.g. 1 a) Do the two lines $\mathbf{x} = (0, 3, -4) + \lambda(1, 0, 2), \mathbf{x} = (5, 1, 0) + \lambda(2, -1, 1)$ intersect & b) if so, find their pt of intersection.

A a) means can you solve

$$(0, 3, -4) + \lambda(1, 0, 2) =$$

for some

b) requires finding

Let's rearrange & rewrite as column vectors

$$\lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \mu$$

Warning: It's very important to remember that 1st column

Elim like Gauss told us to

last column not leading $\Longrightarrow$

For b) 2nd row $\Longrightarrow$

Pt of intersection $\mathbf{x} =$

Rem 1 You can check this by solving for

Rem 2 This method allows you to find pts of intersect of a line & plane too (see §3.8.1 of notes). Coeff matrix has now 3 columns

e.g. 2 Find the Cartesian eqn of the plane $\mathbf{x} = (1, 1, 0) + \lambda(1, 1, -1) + \mu(2, 0, -3)$.

Need find lin condn on x, y, z s.t. you can solve

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda$$

$$\begin{pmatrix} 1 & 2 & x-1 \\ & & \\ & & \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & x-1 & \\ 0 & & & \\ 0 & & & \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & x-1 & \\ 0 & & & \\ 0 & & & \end{pmatrix}$$

can solve iff

This is Cartesian

Rem In Ch.5 will see better method of finding Cartesian form of planes in $\mathbb{R}^3$ using vector & dot product. However, this method works for

Case $A\mathbf{x} = \mathbf{b}$ has unique solns

Terminology An $m \times n$-matrix is square if

e.g.

The diagonal entries of a matrix are those which lie in the i-th row &

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

The $n \times n$-identity matrix I_n is the square matrix

i.e. 0 for every entry except the diagonal

Fact Let A be a square matrix with reduced echelon form U. If every column of U is leading then $A\mathbf{x} = \mathbf{b}$

for any choice of $\mathbf{b}$ & the reduced row echelon form is

There's a matrix C depending only on A s.t. the unique

Reason Clear from any e.g.

e.g. 3 Let $A = \left(\begin{smallmatrix} 2 & 1 \\ 8 & 3 \end{smallmatrix}\right)$

Let's solve $A\mathbf{x} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ for arbitrary

$$\begin{pmatrix} 2 & 1 & & b_1 \\ 8 & 3 & & b_2 \end{pmatrix} \qquad\qquad \begin{pmatrix} 2 & 1 & & 1 & 0 \\ 8 & 3 & & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & b_1 & \\ & & & \end{pmatrix} \qquad \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & & & \end{pmatrix}$$

N.B. 1st 2 columns leading so fact applies.

A square so no row of

$$\begin{pmatrix} 2 & 1 & b_1 & \\ & 1 & & \end{pmatrix} \qquad \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & & \end{pmatrix}$$

$$\begin{pmatrix} 2 & & \\ 0 & 1 & 4b_1 - b_2 \end{pmatrix} \qquad \begin{pmatrix} 2 & & & \\ 0 & 1 & 4 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & \\ 0 & 1 & 4b_1 - b_2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & & \\ 0 & 1 & 4 & -1 \end{pmatrix}$$

Note identity matrix I_2 on left and rows 1 & 2 $\Longrightarrow$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Often replace augmented column in above calcn with

You can easily check the above for any particular value of $\mathbf{b}$.