Lecture 12: Planes in $\mathbb{R}^n$

Aim Lecture Describe & understand

Linear Combinations We start with some motivational discussion.

1-Dim case: Given line $\mathbf{x} =$

All possible "dirn"

2-Dim case: What are all possible

i.e. vectors whose head &

Suppose $\mathbf{v}, \mathbf{w} \neq \mathbf{0}$ lie on plane and not parallel. If $\mathbf{u}$ lies on plane too then picture $\Longrightarrow$

Conversely, $\lambda \mathbf{v} + \mu \mathbf{w}$

Sugggests following concept useful.

Defn Let $\mathbb{F}$ be a field and $\mathbf{v}_1, \ldots \mathbf{v}_m \in$

A linear combination of $\{\mathbf{v}_1, \ldots \mathbf{v}_m\}$ is

The span of $\{\mathbf{v}_1, \ldots \mathbf{v}_m\}$ is

Span $(\mathbf{v}_1, \ldots \mathbf{v}_m) :=$

e.g. 1 For non-zero $\mathbf{v} \in \mathbb{R}^3$

e.g. 2 Show $(5, 3, 2)$ is a lin combn of $(1, 0, 1)$ & $(1, 1, 0)$.

A Need show can write

$(*) \qquad (5, 3, 2) =$

So $(5, 3, 2) =$

Comparing 2nd & 3rd coords $\implies$

Check 1st

Can solve $(*)$ so

Degenerate Span Motivational discussion shows 2 non-zero non-parallel vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ span

Degeneration

Fact Let $\mathbf{v} \in \mathbb{F}^n$ and $\mathbf{w} = \mu\,\mathbf{v}$ for some $\mu \in \mathbb{F}$. Then

If $\mathbb{F} = \mathbb{R}$, then this is a

Proof We need to show that every vector in

i.e. Span

Check Span$(\mathbf{v}) \subseteq$

Check Span$(\mathbf{v}) \supseteq$

The two inclusions yield

Rem This is the typical way of showing 2 sets say, S, T are equal, viz. $S \subseteq T$ & $S \supseteq T$.

Planes in $\mathbb{R}^n$

Defn A plane in $\mathbb{R}^n$ is a set of the form

$S = \{\mathbf{a} + \mathbf{y}|$

for fixed vectors

This is the plane through

Parametric form (or eqn) for this plane is

e.g. 3 Find param form of plane through $A(1,0,0), B(0,1,0), C$

A Pick $\mathbf{a} =$

$\mathbf{x} =$

N.B. There are ∞

e.g. 4 Describe geom

$\{(1,0,0,0)+\lambda(1,-1,0,1)+\mu(-2,2,0,-2)|$

Fact above $\implies$ set is

Planes in $\mathbb{R}^3$ can be described in cartesian coords

Can convert this to param form by

Method 1: Picking 3 points

or Method 2: introduce param λ, μ judiciously. Good choices are

Bad choices occur when

e.g. 5 Find param form of $x + 2y + 3z = 1$.

A Let $\lambda =$

$x =$

$\mathbf{x} =$

e.g. 6 Find param form of $y = z$.

Don't pick $\lambda =$

Pick instead something like

Meaning of λ & μ