Lecture 11: Lines in $\mathbb{R}^n$

Aim Lecture Describe & understand

Parallel 2 non-zero vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ are parallel if

If $\mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ or $\mathbb{R}^2$ this means corresponding geom vectors have same

Line Segments Put coord system on 3-dim (or 2-dim) space.

Rem We often confuse geom vectors with their

$A(a_1, a_2, a_3)$

Define vector $\vec{AB}$:= arrow

$= (b_1$

N.B. $\vec{BA} =$

e.g. 1 $A(1, 2, 3), B(2, 3, 4), C(6, 3, 5), D(4, 1,$

Show $AB \| CD$.

cont'd

Midpoints If A, B are points in space with midpoint M

Proof

e.g. 2 (units = km, kmh^{-1}) Laika is travelling at constant vel $\mathbf{v} = (2, 1, -1)$. If at time

$t = 0$ her initial coords are $\mathbf{a} = (1, 0, 0)$ find her coords when a) time $t = 2$ b) $t = \lambda$.

Lines in $\mathbb{R}^n$

Laika's trajectory is a

Defn A line in $\mathbb{R}^n$ is a subset of the form

$L = \{\mathbf{x} \,|$

for $\mathbf{a}, \mathbf{v}$

$\mathbf{x} = \mathbf{a}+$

λ is called the

Meaning of λ:

N.B. **a** is any

v

Finding Parametric Forms

E.g. 3 Find parametric eqn of line joining points $A(3, 1, 2), B(1, 2, 1)$

Other possible answers are

Cartesian form of line in $\mathbb{R}^2$ is

Obtain from param form by

e.g. 4 Find the cartesian form of the line through $(1, 1)$ parallel to $(2, 3)$.

Param form: $\mathbf{x} =$

$x =$

$y =$

Cartesian eqn is

In 3-dim $ax + by + cz = d$ gives Cartesian eqn of

Need

e.g. 5 find cart form of line in e.g.2.

$\mathbf{x} = (3, 1, 2) + \lambda$

$x =$

$y =$

$z =$

Cartesian eqn is

Converting Cartesian to Parametric form

Harder. Best done after ch. 3 material. Only simple e.g. here.

Key: Introduce parameter λ and write x, y, z interms of λ.

e.g. 6 Find parametric form of

$\frac{x-1}{2} = \frac{y-3}{4}, z = 2.$

Let $\lambda =$

So $x =$

$\mathbf{x} = (x, y, z) =$

Application to classical geometry

Thm Let A, B, C be points in 3-dim space (origin O). The medians are concurrent &

intersect at the centroid which has

Proof.

Median through A is line joining A &

M which has coords

$\vec{AM} =$

so param form of median is

$\mathbf{x} =$

Setting $\lambda =$