Lecture 10: Applications. Laws of Arithmetic for Vectors

Aim of Lecture a) We'll formulate some practical problems using n-tuples.

b) Field axioms gives basic laws for simplifying manipulating numbers.

Let $\mathbb{F}$ = field e.g.

Defn Given $\mathbf{v}, \mathbf{w} \in \mathbb{F}^n$, define the negative of

Define vector subtraction

Relative Velocity Suppose an observer moving

sees an object

Define relative velocity (of object wrt

= velocity object appears to be moving at

e.g.1 Coyote moves 40 kmh^{-1} NE pursuing road runner moving 20kmh^{-1} at 30° east of N. What's apparent speed of

Coords of coords of

Rel vel =

apparent speed =

Line of Best Fit

e.g. 2 Lex believes

i.e. if $x =$ amount of

$y =$ strength of

then $y = mx + b, m < 0$.

Q Formulate precise maths problem to determine

A For $i = 1, \ldots, n$ let

$x_i =$ radiation

$y_i =$

Plot data points

For any given m, b predicted value of y_i is

i-th error in prediction

Define error

$\mathbf{x} =$

$\varepsilon =$

Best choice of

Math formulation: Find m, b which

Will solve in later courses

Laws of Arithmetic for $\mathbb{F}^n$: $\mathbb{F} =$

For vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{F}^n$ and scalars

1. Associative Law of Addition:

2. Commutative Law of Addition:

3. Existence of Zero:

4. Existence of Negatives:

6. Associative Law for Scalar Multn:

7. $1\mathbf{v} =$

9. Scalar Distributive:

10. Vector Distributive:

Cor The above "axioms" hold for

Why?

Some **proofs** of thm: Commutativity of addn seen for geom vectors. In $\mathbb{F}^2$

Let's check 1 other e.g. vector distributive.

In $\mathbb{F}^2$, $\lambda((v_1, v_2)+$

LHS $=$

Can also be "seen" geom using

Like field axioms, laws let you simplify vector expressions.

e.g. 3 $3(4\mathbf{i} + 5\mathbf{j}) - 2(\mathbf{i} + \mathbf{j})$

From the basic laws also follow lots of other useful formulae

Propn For $\mathbf{u}, \mathbf{v}, \mathbf{w}$

a) (cancellation)

b) $0\,\mathbf{v} =$

a) & b) also hold for geometric vectors

Proof a)

b)