

MATH3711: Higher Algebra Test (2006,S1)¹

Provide brief explanations of your answers.

- (4 marks) Let $H = \{1, -1\}$. Is $H \leq \mathbb{R}$? Is $H \leq \mathbb{R}^*$?
- (4 marks) Write $(1\ 3)(2\ 3)$ as a product of disjoint cycles. Is it even or odd?
- (4 marks) What is the subgroup of \mathbb{Z} generated by 4 and 6?
- (2 marks) Let $H \leq G$ with $|G| = 20, |H| = 2$. How many left cosets of H in G are there?
- (6 marks) Consider the function $\phi : \mathbb{R} \longrightarrow \mathbb{C}^* : x \mapsto e^{2\pi ix}$. Show that ϕ is an homomorphism and describe explicitly the isomorphism obtained by applying the first isomorphism theorem to ϕ .
- (6 marks) Let $H := \{M \in GL_n(\mathbb{C}) \mid M = \lambda I, \text{ for some } \lambda \in \mathbb{C}\}$ be the set of scalar matrices in $GL_n(\mathbb{C})$. Show that $M \triangleleft GL_n(\mathbb{C})$. Using the third isomorphism theorem or otherwise, find explicitly, a quotient G of $SL_n(\mathbb{C})$ such that $G \simeq GL_n(\mathbb{C})/H$.
- (4 marks) Write down all the subgroups of $\mathbb{Z}/6\mathbb{Z}$.
- (4 marks) Is the dihedral group D_n for $n \geq 3$ isomorphic to a product of cyclic groups?
- (6 marks) Recall the natural inclusion $D_n \hookrightarrow \text{Perm } \mathbb{R}^2$ defines an action of $G := D_n$ on \mathbb{R}^2 . Let F be a regular n -gon with centre of mass $\mathbf{0}$. Is F a G -stable subset? If s is a vertex of F , what is its G -orbit and what is its stabiliser?

¹by Daniel Chan