School of Mathematics University of New South Wales

MATH3711: Higher Algebra (2007,S1)
Group Theory Revision [1]

1. A cake is divided into 6 equal sectors. In the middle of each candle is placed a red, green or blue candle. How many essentially different ways are there of doing this?

2. In this question, we consider the group G generated by the reflections

$$\tau_1 := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; \tau_2 := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

We let G act on the xy-plane in the usual way (that is, by matrix multiplication).

   (a) Are τ_1, τ_2 symmetries of the square with sides $x = \pm 1, y = \pm 1$?

   (b) Show that $G = \langle \tau_1, \tau_2 \rangle$ is isomorphic to the dihedral group D_4 of order 8. (Hint: what is the rotation $\tau_1 \tau_2$).

   (c) Let G act on $S := \mathbb{R}^2$ by matrix multiplication, i.e. $g.\mathbf{v} := g\mathbf{v}$ for $\mathbf{v} \in \mathbb{R}^2$. Find the fixed point set S^G.

   (d) Let $\mathbf{v} := \left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \right)$. Find the orbit of $\mathbf{v}$.

   (e) Find $H < G$ such that $G.\mathbf{v} \simeq G/H$.

3. Describe explicitly, the 24 elements in the rotational symmetry group of the cube. (Hint: Use the fact that the poles which give the axes of rotation correspond to faces, edges and vertices of the cube).

[1] by Daniel Chan