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# MATH3710: Higher Algebra I, Problem Sheet 5

1. Let $G, H$ be groups. Describe the centre of $G \times H$ in terms of $Z(G)$ and $Z(H)$. Note that this in particular, shows that the product of abelian groups is abelian, a fact which is easily proved directly too.

2. Describe all the subgroups $G$ of $O_2$ of order 16. Your answer should give all subgroups not just subgroups up to isomorphism so in particular, there should be an infinite number of such subgroups. You should probably use the proof of the classification of finite subgroups of $O_2$.

3. Show that the full group $G$ of symmetries of the tetrahedron is $S_4$ as follows. Observe that $G$ embeds naturally in $S_4$. Find a reflection which is a symmetry of the tetrahedron. Hence show that the order of $G$ is 24.

4. Prove using the theorems developed in class that the group of symmetries of a non-square rectangle in $\mathbb{R}^2$ is $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$.

5. Is $D_{14}$ isomorphic to $D_7 \times \mathbb{Z}/2\mathbb{Z}$?

6. Write out the class equation for $D_5$ and $D_6$.

7. Show that the centre of $G = GL_n(\mathbb{R})$ is the set of scalar matrices i.e. $Z(G) = \{\lambda I | \lambda \in \mathbb{R}\}$.

8. Find all the Sylow subgroups of $A_4$.

9. Show that any group $G$ of order 65 is cyclic. As in lectures, you will need to use the Chinese remainder theorem (Q3 of problem sheet 3).

10. Let $G$ be a group of order 42. Use Sylow's theorems to show that $G$ has subgroups of order 14 and 21.

11. Let $G$ be a group of order 8. What can the possible class equations for $G$ be?