Lecture 4: Let's play - Game Theory

Daniel Chan

UNSW

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Conclusion: Sometimes theory doesn't work in practice.

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Q What do they do?

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Prisoner's dilemma example

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Game B

One of your team members bludged and did not contribute towards the project. Do you dob them in?

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- Are you less likely to report a crime if there are more witnesses? (Diffusion of responsibility)
- Is a group of witnesses more or less likely to report a crime the bigger the group is?

Game theory model of crime reporting

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Interpretation: We interpret the Nash equilibrium value of p to be the probability a witness does not report the crime.

Computing the Nash equilibrium

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Conclusion

The Nash equilibrium occurs when

$$g - c = g(1 - p^{n-1}) \Leftrightarrow c = gp^{n-1} \Leftrightarrow p = (c/g)^{1/(n-1)}$$

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