# Fourier series tutorial: Dr Chris Tisdell

www.youtube.com/DrChrisTisdell

Watch my videos and learn:-

- What is a Fourier series?
- How to compute a Fourier series.
- How to simplify the calculations.
- What about convergence?
- Exact values of series (of constants).

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$$a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right)$$

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\, dx$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L}\, dx$$

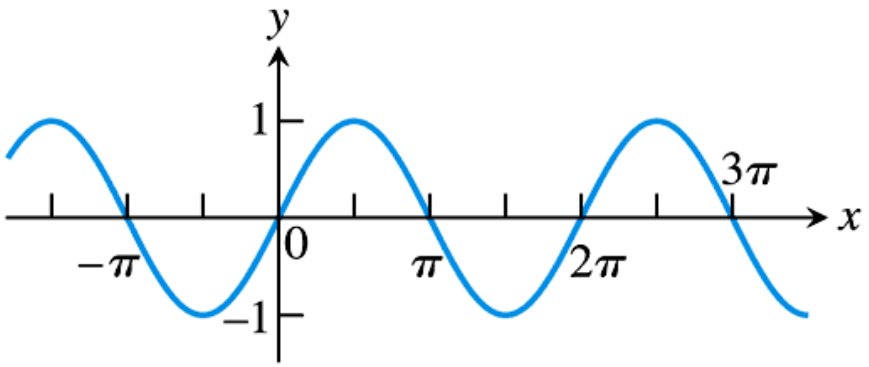
$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L}\, dx$$

## DEFINITIONS Even Function, Odd Function

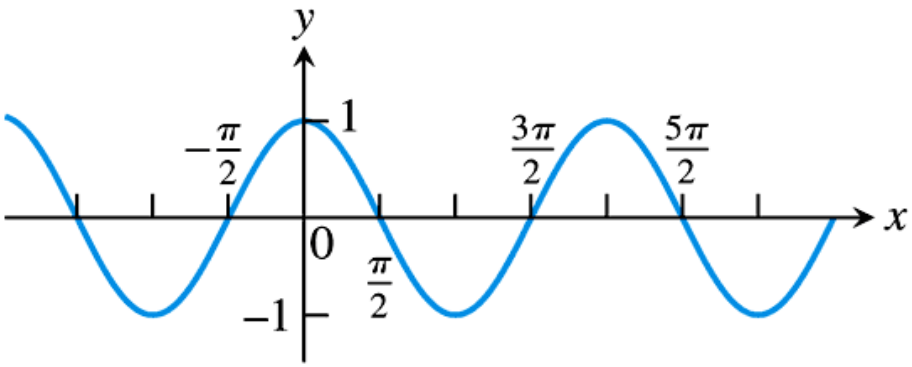
A function $y = f(x)$ is an

$$\textbf{even function of } x \quad \text{if } f(-x) = f(x),$$
$$\textbf{odd function of } x \quad \text{if } f(-x) = -f(x),$$

for every $x$ in the function's domain.




(a) $f(x) = \sin x$




(b) $f(x) = \cos x$

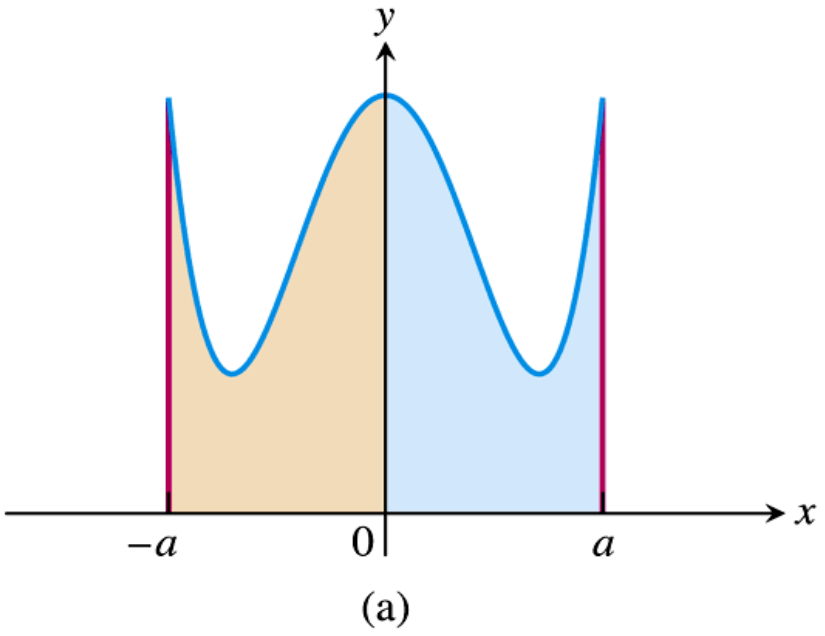
**FIGURE** Graphs of the sine and cosine functions.

## Theorem

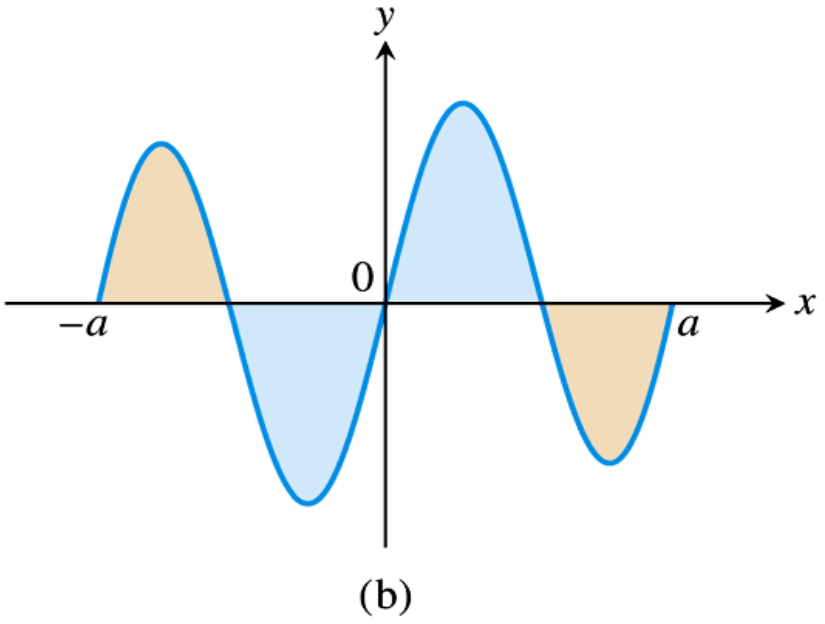
Let $f$ be continuous on the symmetric interval $[-a, a]$.

**(a)** If $f$ is even, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

**(b)** If $f$ is odd, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$.

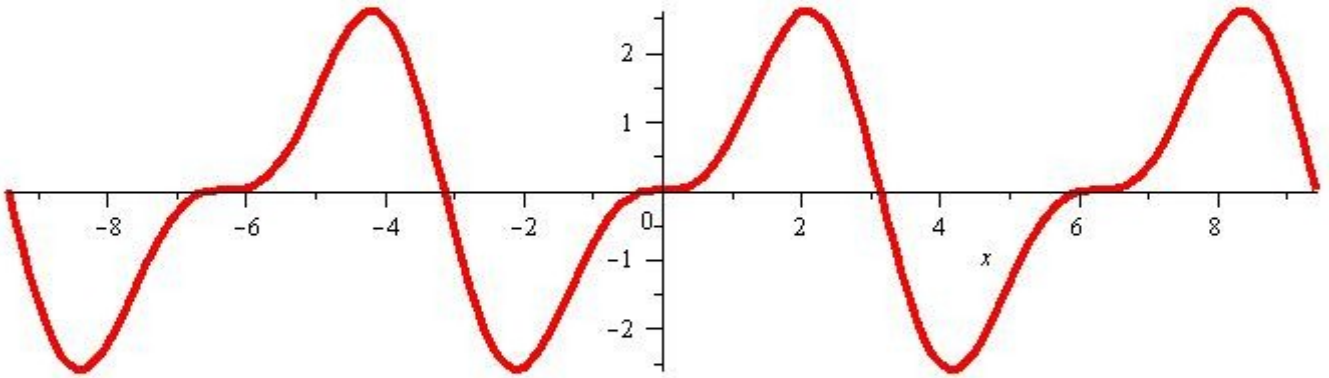


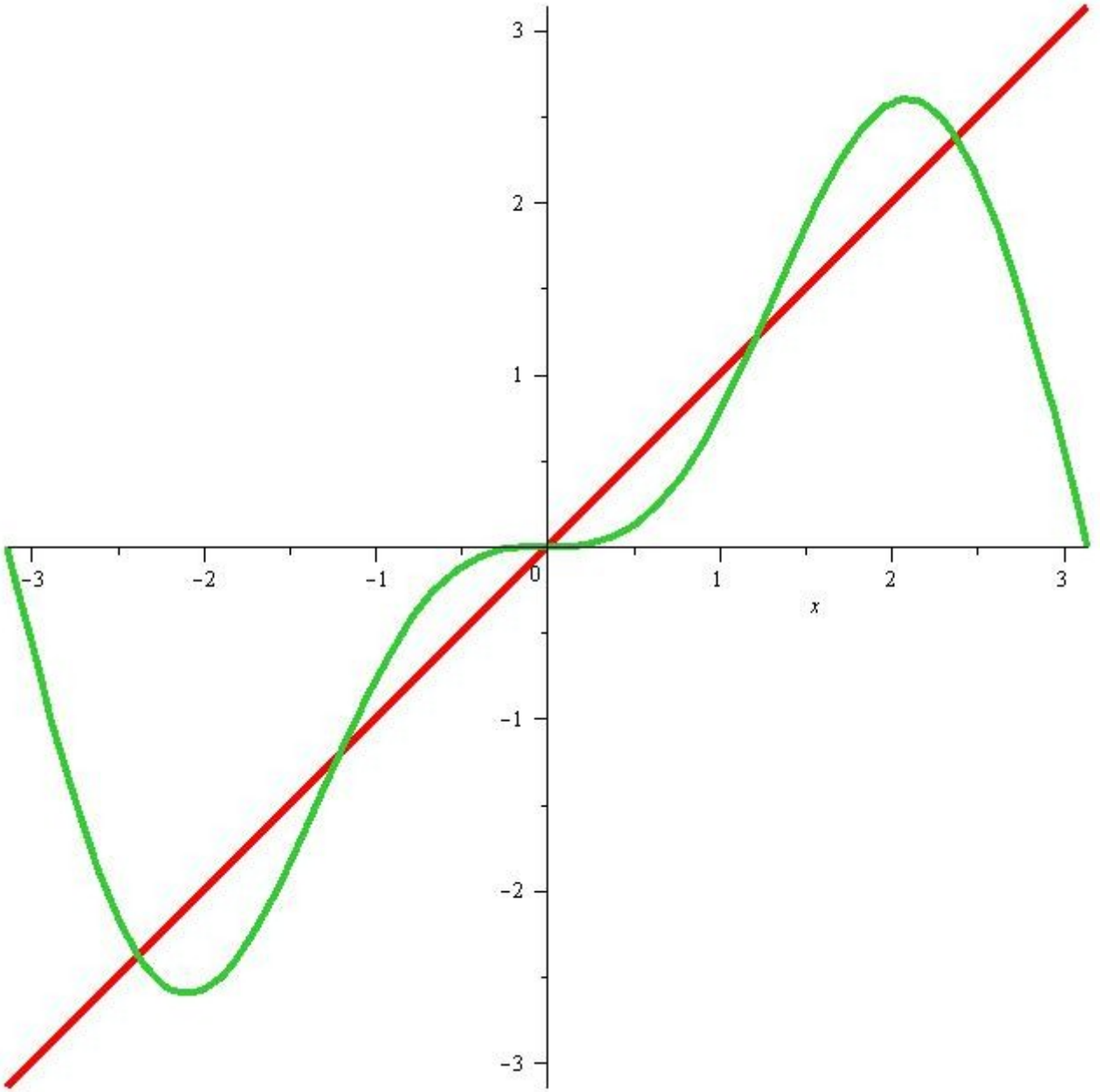

(a)

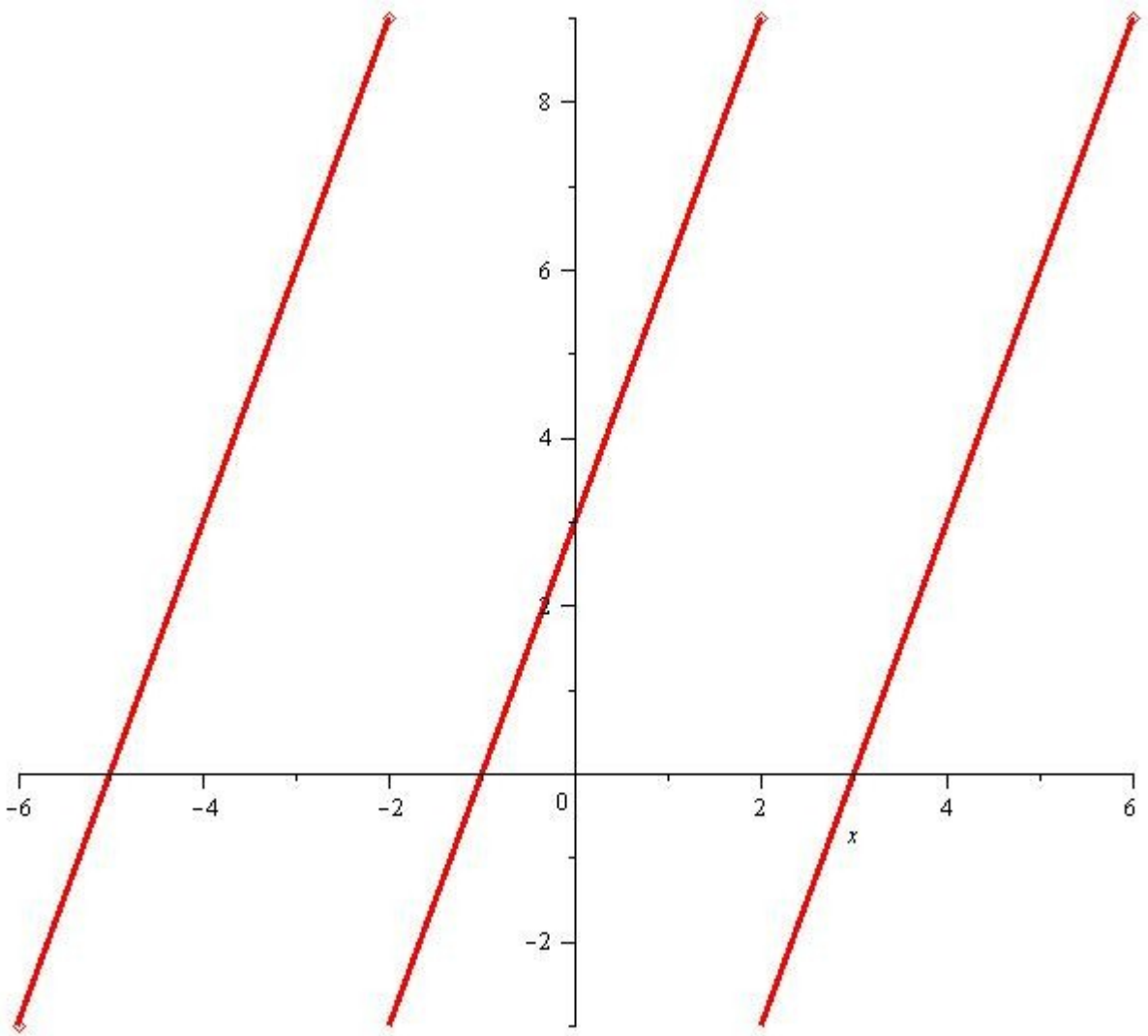



(b)

**FIGURE** (a) $f$ even, $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ (b) $f$ odd, $\int_{-a}^{a} f(x)\,dx = 0$


2
1
0
-1
-2
-8
-6
-4
-2
2
4
6
8
x


3
2
1
-1
-2
-3
-3
-2
-1
0
1
2
3
x


8
6
4
2
-2
-6
-4
-2
0
2
4
6
x

**THEOREM** Let $f(x)$ be a function such that $f$ and $f'$ are piecewise continuous on the interval $[-L, L]$. Then $f$ is equal to its Fourier series at all points where $f$ is continuous. At a point $c$ where $f$ has a discontinuity, the Fourier series converges to

$$\frac{f(c^+) + f(c^-)}{2}$$

where $f(c^+)$ and $f(c^-)$ are the right- and left-hand limits of $f$ at $c$.