Preparing for a PhD

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Should I do a PhD?

- NOT an easy question.
- A PhD is quite different from undergrad, some people strongly prefer one over the other.
- $\sim$ 1 in 3 PhD students seek mental health help.
- $\sim$ 1 in 3 Math PhD students switch to industry/government right after graduating, with further attrition throughout the PhD and after.
- There are consistently less jobs than applicants.
- BUT academia has unique positives. For example, you get to work on cutting edge problems in a collaborating environment.

What are PhD programs looking for?

- In short: Students who are likely to be able to finish their PhD and go on to be successful researchers afterwards.
- Therefore: a strong application is one which convinces admissions you know what a math PhD entails and are likely to succeed at one, through a positive record in related tasks.
- This is also useful for deciding if a math PhD is for you.

Disclaimer: I am a pure maths student. The general gist of the advice should transfer but if you are not in pure maths you might want to also consult someone in your field.

Quick Summary of Applications

You need:

- Transcript: Record of your grades.
- CV: summary of relevant and good stuffs.
- 3 Letters of Recommendation: 3 references, generally professors, write candid letters of their opinions of you.
- (UK/EU/AU): Interview.
- AU: Research Proposal.
- US: GRE: A standardized test on undergrad material.
- US/UK/EU: Statement of Purpose: ≤ 2 pages on why you are applying and are a good candidate.

I think first 4 play the biggest role.
If it is feasible for you, consider applying overseas. There are many strong universities overseas and, if you stay in academia, you will likely have to relocate at least once.

What are they looking for?

Quantitative aspects:

- Grades in relevant coursework.
- Any papers, on arXiv or published. However, in pure maths this is difficult to guarantee.

What are they looking for?

Some examples of qualitative aspects:

- Independence: PhDs require a lot of drive and independent work.
- Deep domain-specific expertise: Research requires an accumulation of knowledge beyond the curriculum. Especially important in UK/EU/AU where you start research immediately.
- "Research" experience: Research is very different from coursework, it is much less clean. Problems are solved on the order of months, not hours.
- "Mathematical maturity": The non-domain specific parts of being a mathematician. Built from doing maths and exposure. Some interesting reading:
  - Terry Tao's blog
  - Advice to a young mathematician

Qualitative aspects are reflected in your letters of recommendation. Letters of recommendation are taken very seriously.

How to develop these aspects?

- In short: *Live mathematics* outside of courses.
- Does not mean go crazy. But you have to see mathematics as extending beyond coursework. Coursework by itself is *NOT* enough.
- There is a clock, and the earlier the better. You develop many of these skills during honours but starting before your honours will make you even more informed.
  There is also a limit to how much you can change these qualitative aspects in a year.
- Also don't be scared of running out of things to do, there will always be more to do. Even if you run out of coursework early there are ways around it.

Read

- Develop a habit of independent reading. This is the first place to start and will inevitably be required.
- Some possible sources for recommendations:
  - Chicago undergraduate math bibliography (slightly dated).
  - Reference request threads on stackexchange, e.g: Abstract algebra books.
  - Course syllabuses. Your professors. More senior students.
- Class notes are a useful supplement. E.g:
  - MIT Courses
  - Many professors post expositions or class notes online. E.g: Pete Clark or Daniel Chan.

Read

Some advice on reading:

- It is *NOT* easy. It is very effortful work. But you get better at it the more you do it.
- "You cannot expect to read mathematics the way you read a novel. If you zip through a page in less than an hour, you are probably going too fast." Possibly hyperbolic but when reading in detail I find it is not far from the truth.
- But it is fundamental for developing the skills necessary to do a PhD. I have found it critical for developing independence and maturity.

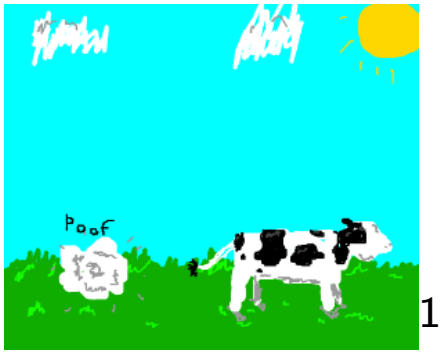
Collaborate

- Find a small group with similar dedication where you are all willing to push each other.
- Decide on something you want to learn, you can ask professors for guidance here. One path is to follow your recent interests.
- Hold each other accountable. Set meeting times and agendas.
- It is important you are comfortable discussing half baked thoughts. You gain a lot from exposure to how other people think.
- You might also be able to get a professor on board. In my experience, many professors are willing to discuss once a week or fortnight with eager students. Worst case they say no.
- Possibly uncomfortable and it may feel pointless at times. But worth it. I see a correlation between when I've learnt the most and when I've been most exposed to outside stimulus.

Engage actively in courses

- Courses are important, they are *necessary* but *not sufficient*.
- Aim to interpret the material from your own perspective. Not necessarily the same as maximizing marks.
- Possible suggestions:
  - Read supplementary texts.
  - Review lecture content after the lecture and make sure it makes sense.
  - Have a zoo of examples you can test general theories on.
  - Relate content to topics you already know.
  - Think up interesting follow up questions that you might then ask at office hours.
  - If you write digital notes, *don't* copy paste. One of the benefits of writing notes is that it forces you to understand the content enough to phrase it in your own terms.

Try different pastures



[1]

New environments usually come with vastly different perspectives on mathematics. This is important for the same reason talking to others is important.

- AMSI Summer School: 1 month intensive programs beyond the curriculum.
- Cross-institutional study: At minimum during honours you can take external courses (Honours Rules).
- Exchange. Many strong partners.
- Other study abroad programs, e.g: BSM.

[1] https://drawception.com/game/AprTXsYGEF/warping-cow/

Trying research

- Science Vacation Research Scholarship: 6 weeks research program.
- Research project during semester. Consider and enquire in advance, there might be preparation required or you may have to show initiative by having an area in mind.
- You can gain experience even if you do not create novel results. Synthesizing research papers and providing a new presentation on known material is valuable experience.
- **Important:** The experience is more important than whether you manage to produce a paper. Especially in pure maths, whether you produce a paper is very luck based and in certain areas very unlikely. The benefit is in the skills you develop. Your development and the work you have put in will be reflected in your advisor's letter.

Trying industry

- If you have doubts, I possibly recommend trying out industry as well through, e.g: an internship or part time.
- I do not think it is really a negative on your application. The more informed you are about doing a PhD the better.

Application specific optimizations

- (US) Do the math subject GRE the year before application. You will have more time to prepare, computations will be fresher in mind and you can retake worst case.
- Think about your letters a year early.
  - The strongest letters come from professors with personal knowledge of your capabilities.
  - Some ways to differentiate yourself: trying to strike up a research project or arrange a reading course or engage deeply in a class and attend office hours.