MATH5825 PROBLEMS, SET 3

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1. Rings and Algebras

Problem 1: Show that
1) the family of intervals

\[ S = \{ [a, b), \ a \leq b \} \]

is a semi-ring.
2) the family of rectangles

\[ S = \{ (a, b) \times [c, d), \ a \leq b, \ c \leq d \} \]

is a semi-ring.

Problem 2: 1) Show that every ring has \( \emptyset \) as its element.
2) Show that every collection \( S \subseteq 2^X \) closed under union and set difference is a ring.
3) Show that not every collection closed under intersection and set difference is a ring.
4) Show that not every collection closed under intersection and union is a ring.
5) Show that a \( \sigma \)-algebra is closed under countable intersection.

Problem 3: Let \( S \) be a semi-ring and let

\[ R = \left\{ \bigcup_{i=1}^{n} A_i, \ A_i \in S \right\} \quad \text{and} \quad \hat{R} = \left\{ \bigcup_{i=1}^{\infty} A_i, \ A_i \in S \right\} \]

Show that \( R \) is a ring whereas \( R_\sigma \) is in general not.

Problem 4: Let \( A \subseteq X \) be a non-empty subset. Describe the \( R_\sigma(\{A\}) \).

Problem 5: Describe the Borel ring and the Borel \( \sigma \)-ring on the line.

Problem 6: Show that every semi-ring is a ring.

References


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1Added in this version.