Lecture 17: Mean and Variance for Discrete Random Variables

**Aim Lecture** Define and compute the mean & variance for a discrete random var.

Recall for random var $X$ on a population of size $n$, with values $x_k$ occurring with freq $f_k$ then,

$$
\bar{x} =
$$

Since $\frac{f_k}{n}$

**Defn** (Expected Value) For $X = \text{discrete random var with}$

values

& probability distribn
the mean or
\[ \mu = E(X) = \]

whenever the sum converges.

**E.g. 1** Uniform distribn on \( \{1, \ldots, n\} \).

\[ p_1 = \ldots \]

\[ E(X) = \]

Given a random var \( X \) and a fn \( g : \mathbb{R} \rightarrow \mathbb{R} \)
we can consider a new random var \( Y = g(X) \).

**E.g. 2** \( X = \) radius of

\( Y = \text{vol} \)

\[ = \]

**Propn 1** Let \( X \) be a discrete random var
with

Let $g : \mathbb{R} \to \mathbb{R}$ be a fn and $Y$
Then $E(Y) =$
Proof: omitted.

**Propn 2**
i) For $a, b \in \mathbb{R}$,
$E(aX + b) =$

ii) For fns $g, h : \mathbb{R} \to \mathbb{R}$,
$E(g(X) + h(X)) =$

Proof: i) Propn 1 $\implies$
$E(aX + b) =$

$= a \sum$
ii) Sim. omitted.

E.g. 3 Let $X =$ temp of Sydney in Celsius
$Y =$ temp of Sydney in
$E(Y) = E($

Recall for random var $X$ on a population of size $n$, with values $x_k$ occurring with freq $f_k$
then the sample variance is

This suggests as before

Defn (Variance) For $X =$ discrete random var with values $\{x_k\}$ & probability distribn $\{p_k\}$, the
$\text{Var}(X) :=$
The standard deviation is

\[ \sigma(X) = \]

Cor Var(X) = \( E(X^2) - E(X)^2 \).

Proof: Var(X) = \( E((X - E(X))^2) \)

\textbf{N.B.} In gen, you expect \( E(X^2) \neq \)

\textbf{Generating Functions}

The following method gives best way to find \( E(X), \) Var(X) if values of X are \( x_k = k. \)
Defn For a probability distrbn \( \{p_k\} \), its generating fn is

\[ E.g. 1 \quad \text{Uniform distrbn on } \{0, 1, 2\}. \]
\[ p_0 = \]
\[ p(t) = \]

Propn 3 Let \( p(t) \) be the generating fn for the prob distrbn for \( X \) with values \( x_k = k \).
Then
\[ a. \ E(X) = \]
\[ b. \ E(X^2 - X) = \]
\[ c. \ E(X^2) \]
Proof: a) \( p'(t) = \]
\[ p'(1) = \]
b) \( p''(t) = \)

\[ p''(1) = \]

c) \( E(X^2) = E(X^2 - X + X) \)

We’ll use this propn to compute \( E(X) \) & \( \text{Var}(X) \) for the Poisson & Binomial distrbn.

**Poisson Distrbn** with parameter \( \lambda \)

Recall \( p_k = \)

\[ p(t) = \]

**Propn 4** For the Poisson distrbn,
a. $E(X) = \lambda$ 

Proof: 

a) $p'(t) =$

$E(X) =$

b) $p''(t) =$

$E(X^2) =$

$\text{Var}(X) =$

\textbf{Binomial Distrbn} $n \in$

Recall $p_k = B(n, p, k) =$

Generating fn $p(t) =$

\textbf{Propn 5} For the Binomial distrbn,
a. $E(X) = np$ (You know this!)

b. $Var(X) = np(1 - p)$.

Proof: a) $p'(t) =$

$E(X) =$

b) Sim. Exercise.

**E.g. 4** If $X$ has Poisson distrbn with param $\lambda = 3$. What’s the probability that $X$ is below average?

Ans: Average $E(X) =$

$P(X <$ 

$( \approx .423)$
**E.g. 5** Suppose a biased coin is tossed 64 times.

Let $X =$ no. heads

$p_k =$

If sample variance is 12, what’s the probability of all heads showing up assuming sample variance is actual variance?

$\text{Var}(X) = 12 =$

Solving for $p$ gives $p =$

$\therefore \text{prob} = p_{64} =$
E.g. 6 Let $X$ be the number of times Gollum says “my precious” in any hour. $X$ has Poisson distribution with $E(X) = 2$. Is the expected value greater than the median value?

$p_k =$

$p_0 + p_1 =$

$(\approx .406)$

$p_0 + p_1 + p_2 =$

$(\approx .677)$