Lecture 1: Vector Spaces

Aim of Today’s Lecture:

Properties of $\mathbb{R}^n$: Write $V = \mathbb{R}^n$, $F = \mathbb{R}$.

For any $u, v, w \in V$

1. Closure Under Addition:

2. Associative Law of Addition:

3. Commutative Law of Addition:

4. Existence of Zero:
5. Existence of Negatives:

6. Closure Under Scalar Multiplication:

7. Associative Law for Scalar Multn:

8. $1v =$

9. Scalar Distributive:

10. Vector Distributive:

**Defn of Vector Space**

Let $\mathbb{F}$ be a field like $\mathbb{Q}, \mathbb{R}, \mathbb{C}$. A vector space over $\mathbb{F}$ is
a. A set $V$ of

& b. An addition law denoted $+$ which

This new vector is called

& c. A scalar multn law

satisfying

Properties 1-10 called

Eg Set $\mathbb{R}^n$ with

addition rule:

scalar multiplication rule:
$\mathbb{C}^n$ is a vector space over
Define addn law as usual

Axioms

$\textbf{Eg}$ Let $M_{mn}(\mathbb{F})$ be the set of $m \times n$-matrices over the field $\mathbb{F}$. Define

vector addition:

scalar multiplication:

You can check all axioms. Here we’ll only check axiom 4:

$\textbf{Eg}$ Let $\mathbb{P}$ be set of real polynomials.
\( p(x) = \)
\( q(x) = \)
Vector addn: \((p + q)(x) = \)

Scalar Multn: For \( \lambda \in \) 

Can check 10 axioms to 

\( \lambda(p(x) + q(x)) = \)

Why vector spaces? 
Vector spaces exhibit similar 

Eg For vectors \( \mathbf{v}, \mathbf{w} \in \mathbb{R}^n \) simplify \( 2 \mathbf{v} + \mathbf{w} - 3 \mathbf{v} \).
Eg For $m \times n$-matrices $M, N$ simplify $2M + N - 3M$.

Only used

Conclusion: Much of the arithmetic for geometric vectors

**Properties of Vector Spaces**

**Propn** In a vector space $V$

1. Uniqueness of Zero: The eqns in $w$

(*)

has a unique soln denoted $0$ & called
2. Cancellation:

3. Uniqueness of negatives: For any \( \mathbf{v} \in V \),
the eqn

We call

Proof: Do 1 only. Soln exists by axiom.
Suppose \( \mathbf{w} = \)

\[ \text{Eg } M_{mn}(\mathbb{F}). \text{ The unique negative of } A \]

\[ \text{Propn} \text{ Let } V \text{ be a vector space over } \mathbb{F}. \text{ For} \]
$\lambda \in \mathbb{F}, \mathbf{v} \in V$

1. $\lambda \mathbf{0} =$
2. $0 \mathbf{v} =$
3. $(-1) \mathbf{v} =$
4. $\lambda \mathbf{v} = \mathbf{0} \implies$

Proof: Do 2 only.

$0 \mathbf{v} + 0 \mathbf{v} =$

Cancel

\textbf{e.g.} Twisted $\mathbb{C}^n$. $V =$

Addn:

New twisted scalar multn:

$V$ is a vector

Why?