Lecture 7: Trigonometric identities from complex numbers

**Aim Lecture** Euler’s formula suggests

**Binomial Formula**

**Defn-Propn** For $n \in \mathbb{N}, 0 \leq$

the binomial

It’s the no. ways of picking

**Binomial Thm**

$$(a + b)^n =$$
Why?

Facts 1. $e^{i\theta} =$
2. $\sin \theta =$
3. $\cos \theta =$

Proof From picture

2. (3. is similar)

e.g. 1 Write $\sin^4 \theta$ in terms of $\cos$

A $\sin^4 \theta =$
\[ = \frac{1}{4}(e^{i4\theta} - 4e^{i3\theta} e^{-i\theta} + \ldots) \]

N.B. \( \sin^4 \theta \) is an even

**Uses** Can integrate

**Rem** Fourier theory (taught in 2nd yr) shows that any nice fn of period \( 2\pi \) can be expanded as above using constants, \( \cos \theta \),

Conversely, \( \sin n\theta \), \( \cos n\theta \) can be converted back to a polynomial
e.g. 2 Write \( \sin 3\theta, \cos 3\theta \)

A De Moivre \( \implies \)

\[
\cos 3\theta + i \sin 3\theta
\]

Equate

\[
\cos 3\theta =
\]

\[
\sin 3\theta =
\]

The answer is not

\[
\cos 3\theta = \cos^3
\]
Bizarre Application

Solve $4z^3 - 3z = \frac{\sqrt{3}}{2}$

A Suppose there is a soln of form $z = \cos \theta$

So $4z^3 - 3z = \cos \theta$, $\frac{\sqrt{3}}{2}$ =

Hence $3\theta$ =

See algebra notes Ch 1, ex.61 for more info. Method generalises to higher order poly if you use “elliptic” fns.

Sums of trig fns

e.g. 3 Find

$\Sigma := \cos \theta + \cos 3\theta + \cos (2n + 1)\theta$.

A $\Sigma$ =

But the sum of a
\[ e^{i2\theta} - 1 = \]

\[ e^{i(2n+3)\theta} \]