Lecture 6: Solving polynomial equations

**Aim Lecture** The use of polar forms makes it

**Square Roots**

e.g. 1 Solve $z^2 = -5 + 12i$

A Let $z = a +$

Equate real & imag parts

Solve simultaneously by guessing or elim $a$

or better still
Quadratic eqn Quadratic formula still holds.

e.g.2 Solve $z^2 + (-4 + i)z + (5 - 5i) = 0$. 

quadratic formula:

discriminant = 

From e.g.1 we see

Hence,
Roots of complex numbers

e.g.3 Find all cube roots of $8i$.

A Suppose $z^3 = 8i, \ z = r$

Polar form of $8i =$

$z^3 = r$

Equate moduli:

Equate arg:

N.B. $-\pi < \theta \leq \pi \implies$

Hence,
So \( z = \)

Note: 3 roots. In lecture 8 we will see

Let’s plot the 3 cube roots

**Cardano’s Formula** for the cubic eqn

\[
z^3 + az + b = 0
\]

is \( z = u - \frac{a}{3u} \) where

\[
u = \left[ -\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}} \right]^{1/3}
\]
Proof Suppose $z = u - v$

Case $b = 0$